

۱۴، ۷۵

کلاس: دهم دفتر B

تاریخ شماره: ۱۴

موضوع: هندسه

۱- الف) $\frac{1}{r}, 1, r, \dots \rightarrow q = r$ $a = \frac{1}{r}$ $t_{10} = aq^9$ $\frac{1}{r} \times r^9 = r^8 = 256$

ب) $a_{17} = aq^{16}$ $a_{13} = aq^{12}$ $\frac{a_{17}}{a_{13}} = \frac{aq^{16}}{aq^{12}} = q^4 = r^4 = 16$

ج) $a_4 = aq^3 = \frac{1}{r} \times r^3 = 8$ $a_{10} = aq^9 = \frac{1}{r} \times r^9 = 256$ $\frac{256 + 8}{r} = 13$
 $a_1 \times a_8 = a_n \times a_n \rightarrow a_n = a_1 \times q^7 = \frac{1}{r} \times r^7 = r^6 = 32$
 $a_n = 121 = a \times q^{n-1}$ $256 = r^{n-1} = r^7 \Rightarrow n = 8$

د) $a_5 = aq^4 = 12$ $a_8 = aq^7 = 96$ $\frac{a_8}{a_5} = \frac{aq^7}{aq^4} = \frac{96}{12} = q^3 = 8 \Rightarrow q = 2$
 $a \times r^4 = 12$ $a = \frac{12}{r^4} = \frac{3}{r}$
 $a_{10} = aq^9 = \frac{3}{r} \times r^9 = 3 \times r^8 = 384$

۳- الف) $a_1 \times a_1 q \times a_1 q^2 \times a_1 q^3 \times a_1 q^4 = a_1^5 \times q^{10} = 256 = 2^8 = (a_1 q^2)^5 \Rightarrow a_1 q^2 = 4$
 $a_3 = aq^2 = 4$

ب) $a_1 \times a_5 = a_1 \times a_1 q^4 = a_1^2 q^4 = (a_1 q^2)^2 = 4^2 = 16$

۴- $r^a, \sqrt{r^b}, r^b$ $\frac{r^{\frac{a+b}{2}}}{r^a} = \frac{r^b}{r^{\frac{a+b}{2}}} = q = r^{\frac{a+b}{2}-a} = r^{\frac{b-a}{2}} \Rightarrow \frac{a+b}{2} - a = \frac{b-a}{2} \Rightarrow a+b = b-a$
 $\frac{a+b}{2} = \frac{b-a}{2} \Rightarrow r^{\frac{a+b}{2}} = r^{\frac{b-a}{2}}$

۵- $a_3 = aq^2 = 1-x$ $a_7 = aq^6 = x$ $a_{11} = aq^{10} = x+1$
 $a_3 a_{11} = a_7^2$ $(1-x)(1+x) = x^2 \Rightarrow 1-x^2 = x^2$ $1 = 2x^2$ $\frac{1}{2} = x^2$ $x = \pm \sqrt{\frac{1}{2}}$

$\frac{-\sqrt{\frac{1}{2}}}{1 + \sqrt{\frac{1}{2}}} = q^2 \rightarrow x$

اگر $x = -\sqrt{\frac{1}{2}}$ باشد q متناقص می‌شود

$$a + aq^n = r \quad a(1 + q^n) = r$$

$$\frac{a(1+q)(1-q+q^2)}{aq(1+q)} = \frac{r_1}{r} = 1$$

$$\Sigma \circ q + 12q^2 = 12 \quad \Sigma q(10 - 3q) = 12 = 3 \times \Sigma \Rightarrow q = 3 \Rightarrow a_1 = 1 \Rightarrow a_1 q^3 = 1 \times 3^3 = \underline{27}$$

$$a \otimes a q = 1^{\otimes} \quad a \otimes a q = 1^{\otimes} = a^{\otimes} q^{\otimes} \Rightarrow a q = \sqrt{2} \quad a = \sqrt{2}, 1^{\otimes} \quad a = 1^{\otimes}, \sqrt{2} = 1^{\otimes}$$

$$S_n = a_1 \left(\frac{q^n - 1}{q - 1} \right) \quad q = r = \frac{a}{r} \quad a_1 = r \quad r \left(\frac{r^{10} - 1}{r - 1} \right) = r^{10} - 1 \quad \text{الف (1-10)}$$

$$r_x r_x r_x r_x r_x r_x r_x \dots \times \frac{r_x r_x^{n-1}}{r_x r_x^q} \Rightarrow r_x^1 \cdot r_x^{\frac{q \times 1}{q}} = r_x^1 \cdot r_x^1 = r_x^2$$

$$\begin{array}{ccccccc}
 a, & aq, & aq^r, & aq^{r^2}, & aq^{r^3}, & \dots & aq^{r^{n-2}}, & aq^{r^{n-1}} \\
 \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & & \downarrow \\
 a(q-1) & aq(q-1) & aq^r(q-1) & aq^{r^2}(q-1) & aq^{r^3}(q-1) & & aq^{r^{n-2}}(q-1) & aq^{r^{n-1}}(q-1)
 \end{array}$$

مساعده مسعود $a(q-1)$ نسبت به q صحیح است و q ضرب بر مسعود
 همیشه فکر نسبت q است

$$S_n = a + (n-1)d + a + (n-2)d + a + (n-3)d + \dots + a + d + a$$

ب- الف)

$$S_n = a + a + d + \dots + a + (n-r)d + a + (n-1)d$$

$$1S_n - 1a + (n-1)d + 1a + (n-1)d + \dots = n(1a + (n-1)d) \Rightarrow S_n = \frac{n}{2} (1a + (n-1)d)$$

$$ar^{n-1} + ar^{n-2} + \dots + ar + a = S_n$$

$$a_1q^n + \underbrace{a_1q^{n-1} + \dots + a_1q^1 + a_1q^0}_{S_{n-1}} = q S_n$$

$$a_1 q^n + S_{n-1} = q S_n \Rightarrow S_n (q-1) = a_1 (q^n - 1) \Rightarrow S_n = a_1 \times \frac{q^n - 1}{q - 1}$$

سوال ۲

$$\begin{aligned} a + aq^r &= 11 \\ aq + aq^r &= 12 \end{aligned} \rightarrow \frac{a(1+q^r)}{aq(1+q)} = \frac{(q^r - q + 1)}{q} = \frac{1}{3}$$

$$3q^r - 3q + 3 = 12q \rightarrow 3q^r - 12q + 3 = 0 \xrightarrow{\text{بسط}} q^r - 4q + 1 = 0$$

$$q = \frac{4}{3} \leq q = \frac{1}{3}$$

$$q = 3 \rightarrow a(1+27) = 11 \rightarrow a < 1 \rightarrow 1, 3, 9, 27$$

$$q = \frac{1}{3} \rightarrow \frac{a}{3} + \frac{a}{9} = 12 \rightarrow \frac{4a}{9} = 12 \rightarrow a = 27$$

$$27, 9, 3, 1 \rightarrow \text{max مجلد} \rightarrow 27$$

سوال ۷

$$a_1 = \frac{a}{9}$$

$$a_2 = a$$

$$a_3 = aq$$

$$\frac{a}{9} \times a \times aq = 27 \rightarrow a^3 = 27 \rightarrow \underline{a = 3}$$

$$\text{if } a = 3 \rightarrow \frac{3}{9} + 3 + 3q = 12 \rightarrow 3q^r - 12q + 3 = 0$$

$$\Delta \rightarrow \frac{1 \pm \sqrt{1 - 44}}{4} = \frac{1 \pm 1}{4} \rightarrow \begin{matrix} 3 \\ \frac{1}{3} \end{matrix}$$

$$q = 3 \rightarrow \text{مجدد} = 1, 3, 9$$

$$q = \frac{1}{3} \rightarrow \text{مجدد} = 9, 3, 1$$