نام و نام خانوادگی ..... پاسخنامه تشریحی تکلیف شماره ۱۰۴ ... کلاس .....B.....

۱

الف) $a_n = \frac{1}{2}, 1, 2, \dots \quad a_n = \frac{1}{2} \times 2^{n-1} \Rightarrow a_{10} = \frac{1}{2} \times 2^9 = \boxed{256}$

ب) $\frac{a_{17}}{a_{12}} = \frac{aq^{16}}{aq^{12}} = q^4 = 2^4 = \boxed{16}$

ج) $\frac{aq^3}{aq^9} = q^4 = 2^4 = \boxed{64}$

د) $\frac{1}{2} \times 2^{n-1} = 128 \Rightarrow 2^{n-1} = 256 \Rightarrow n-1 = 8 \Rightarrow \boxed{n=9}$

۲

$\left.\begin{matrix} t_5 = aq^4 = 12 \\ t_8 = aq^7 = 96 \end{matrix}\right\} q^3 = 8 \Rightarrow q = 2 \quad aq^4 = a \times 16 = 12 \Rightarrow a = \frac{3}{4}$

$\frac{3}{4} \times 2^{n-1} \Rightarrow \frac{3}{4} \times 2^9 = \boxed{384}$

۳

$\underbrace{a_1 \times a_2 \times a_3 \times a_4 \times a_5}_{a_3 \times a_3} = 243$

$a_0 \times a_1 = a_2 \times a_2$

الف) $(a_3)^5 = 243 \Rightarrow \boxed{a_3 = 3}$

ب) $a_1 \times a_5 = a_2 \times a_4 = a_3 \times a_3 \Rightarrow 3 \times 3 = \boxed{9}$

۴

$(4\sqrt{2})^2 = 2^b \times 2^a \Rightarrow 32 = 2^b \times 2^a \Rightarrow 32 = 2^{a+b}$

$2^a, \boxed{4\sqrt{2}}, 2^b$ (واسطه)

$32 = 2^5 \Rightarrow 2^{a+b} = 2^5 \Rightarrow a+b = 5$

$\frac{a+b}{2} = \frac{5}{2} = \boxed{2,5}$

۵

$\frac{t_5}{t_2} = \frac{t_{11}}{t_5} \Rightarrow \frac{x}{1-x} = \frac{x+1}{x} \Rightarrow x^2 = 1 - x^2 \Rightarrow 2x^2 = 1 \Rightarrow x^2 = \frac{1}{2}$

$x = \pm\sqrt{\frac{1}{2}} \quad \boxed{\pm \frac{1}{2}}$

$t_1 + t_4 = 28 \Rightarrow a + aq^3 = 28 \Rightarrow a(1+q^3) = 28$
$t_2 + t_3 = 12 \Rightarrow aq + aq^2 = 12 \Rightarrow aq(1+q) = 12$

$\frac{a(1+q^3)}{aq(1+q)} = \frac{28}{12} \Rightarrow \frac{(1+q)(1-q+q^2)}{q(1+q)} = \frac{7}{3}$

$\frac{1-q+q^2}{q} = \frac{7}{3} \Rightarrow 3-3q+3q^2 = 7q \Rightarrow 3q^2-10q+3=0$

$q = 10 \pm \sqrt{64} \Rightarrow \frac{10 \pm 8}{6} = 3, \frac{1}{3} \Rightarrow \boxed{q=3}$

۱، ۳، ۹، ۲۷ $\boxed{27}$

۶

$a_1 + a_2 + a_3 = 13 \Rightarrow a + aq + aq^2 = 13 \Rightarrow a_1(1+q+q^2)$

$a_1 \times a_2 \times a_3 = 27 \Rightarrow a \times aq \times aq^2 = 27 \Rightarrow (a_1 q)^3 = 27 \Rightarrow a_1 q = 3 \Rightarrow a_2 = 3$

$a_1 + a_2 + aq^2 \Rightarrow a(1+q^2) = 10 \Rightarrow a + aq^2 = 10$
$a_1 \times a_2 \times a_3 \Rightarrow (aq)^3 = 9 \Rightarrow a^2q^2 = 9$ $\Big\} a = 1$

$\boxed{1, 3, 9}$

۷

$a_1 = 2 \qquad q = \frac{a_2}{a_1} = \frac{6}{2} = 3$

الف) $S_{10} = 2\left(\frac{3^{10}-1}{3-1}\right) = 2\left(\frac{3^{10}-1}{2}\right) = 3^{10} - 1 \Rightarrow 59049 - 1 = \boxed{59048}$

ب) $p = (\text{اولی} \times \text{آخری})^5 = (2 \times 2 \times 3^9)^5 = \boxed{2^{10} \times 3^{45}}$

$aq^9 = 2 \times 3^9 = 19683$

۸

$\frac{aq(q-1)}{a(q-1)} = q \Rightarrow \frac{aq^2-aq}{aq-a} = \frac{t_2-t_1}{t_2-t_1}$

$\frac{aq^2(q-1)}{aq(q-1)} = q \Rightarrow \frac{aq^3-aq^2}{aq^2-aq} = \frac{t_4-t_3}{t_3-t_2}$

$\underbrace{a}_{q}, \underbrace{aq}_{q}, \underbrace{aq^2}_{q}, aq^3$

۹

$S_n = a + (a+d) + (a+2d) + \cdots + (a+(n-1)d)$
$S_n = (a+(n-1)d) + (a+(n-2)d) + \cdots + a$

$2S_n = [a+(a+(n-1)d)] + [(a+d)+(a+(n-2)d)] + \cdots + [(a+(n-1)d)+a]$

$\Rightarrow 2S_n = [2a+(n-1)d] + [2a+(n-1)d] + \cdots + [2a+(n-1)d] \Rightarrow$

$2S_n = n[2a+(n-1)d] \Rightarrow S_n = \frac{n}{2}[2a+(n-1)d]$

۱۰