آمادگی دهم تجربی — تکلیف ۱۴

سوال ۱)

$a_n = \frac{1}{2}, 1, 2$ (×2, ×2) $\quad a_n = \frac{1}{2} \times 2^{n-1}$

الف) $a_{10} = \frac{1}{2} \times 2^{10-1} = 256$ ✓

ب) $\frac{a_{17}}{a_{13}} = \frac{\frac{1}{2} \times 2^{17-1}}{\frac{1}{2} \times 2^{13-1}} \Rightarrow \frac{\frac{1}{2} \times 2^{16}}{\frac{1}{2} \times 2^{12}} = 2^4 = 16$ ✓

ج) $b^2 = ac \quad t_4, \ldots, t_{10}$

$t_4 = \frac{1}{2} \times 2^{4-1} = 4 \qquad t_{10} = \frac{1}{2} \times 2^{10-1} = 256$

$b^2 = 4 \times 256 \rightarrow b^2 = 1024 \Rightarrow b = \sqrt{1024} = 32$ ✓

$\frac{1}{2} \times 2^{n-1} = 128 \rightarrow 2^{n-1} = 128 \times 2 \Rightarrow$

$2^{n-1} = 256 \rightarrow n = 9$ ✓

سوال ۲)

$t_5 = 12 \quad t_1 r^4 = 12$

$t_8 = 96 \quad t_1 r^7 = 96$ $\Big\}$ $\frac{r^7}{r^4} = \frac{96}{12}$

$\Rightarrow r^3 = 8 \rightarrow r = 2$

$t_5 \Rightarrow t_1 \times 2^4 = 12 \rightarrow t_1 = \frac{3}{4}$ $\Big\}$ $t_{10} = t_1 \times r^9 = \frac{3}{4} \times 2^9$

$\Rightarrow t_{10} = 3 \times 128 = 384$ ✓

سوال ۳)

$a_1 \times a_1r^1 \times a_1r^2 \times a_1r^3 \times a_1r^4 = 243$

$a_1^5(r^1 \times r^2 \times r^3 \times r^4) = 243$

$a_1^5 r^{10} = 243 \qquad a_1r^2 = a_3 \Rightarrow a_3^5 = 243$

الف) $a_3^5 = 3^5 \longrightarrow a_3 = 3$ ✓

ب) $a_1 \times a_5 = a_1 \times a_1r^4 = a_1^2r^4 \rightarrow a_1r^2 = 3 = a_3$

$a_1^2r^4 = (a_1r^2)^2 = 3^2 = 9$ ✓

سوال ۴)

$(4\sqrt{2})^2 = 2^b \times 2^a \longrightarrow 16 \times 2 = 2^{b+a}$

$32 = 2^{b+a} \longrightarrow 2^5 = 2^{b+a}$

$b + a = 5 \longrightarrow$ واسطه حسابی $\Rightarrow \frac{a+b}{2} = \frac{5}{2}$ ✓

سوال ۵)

$a_3 = 1 - x \longrightarrow a_1r^2 = 1 - x$

$a_7 = x \longrightarrow a_1r^6 = x$

$a_{11} = 1 + x \rightarrow a_1r^{10} = x + 1$

$\frac{a_7}{a_3} = \frac{a_1r^6}{a_1r^2} \Rightarrow r^4 = \frac{x}{1-x}$ ، $\frac{a_{11}}{a_7} = \frac{a_1r^{10}}{a_1r^6} \Rightarrow r^4 = \frac{1+x}{x}$

$\frac{x}{1-x} = \frac{1+x}{x} \longrightarrow 2x = 1 - x^2$

$2x^2 = 1 \longrightarrow x^2 = \frac{1}{2} \Rightarrow x = \pm\frac{1}{\sqrt{2}}$

$-\frac{1}{\sqrt{2}}$ را نمی‌توان جز xها حساب کنیم چون r^4 منفی نمی‌شود مگر اینکه اعداد منفی باشند

$\Rightarrow x = \frac{1}{\sqrt{2}}$ ✓

سوال ۶)

$t_1 + t_4 = 28$, $t_2 + t_3 = 12$ → $\begin{cases} t_1 + t_1 r^3 = 28 \\ t_1 r + t_1 r^2 = 12 \end{cases} = \dfrac{t_1(1+r^3)}{t_1 r(1+r)} = \dfrac{7}{3}$

اتحاد چاق و لاغر ← $\dfrac{1+r^3}{r(1+r)} = \dfrac{7}{3} \rightarrow \dfrac{(1+r)(1-r+r^2)}{r(1+r)} = \dfrac{1-r+r^2}{r} = \dfrac{7}{3}$

$3(1-r+r^2) = 7r \rightarrow 3 - 3r + 3r^2 = 7r$

$3r^2 - 10r + 3 = 0 \rightarrow \Delta = 100 - 36 = 64$

$r = \dfrac{10 \pm 8}{6} \rightarrow r = 3 \quad r = \dfrac{1}{3} \quad a = 1$

$a_1 \xrightarrow{r=3} 1 \quad a_2 \rightarrow 3 \quad a_3 \rightarrow 9 \quad a_4 \rightarrow 27$

$r = \frac{1}{3}$: $27 \quad 9 \quad 3 \quad 1$

★ بزرگترین جمله ۲۷ است در $r=3$, $r=\frac{1}{3}$ — (۴)

سوال ۷)

$t_1 + t_2 + t_3 = 13 \rightarrow \begin{cases} t_1 + t_1 r + t_1 r^2 = 13 \\ t_1(1+r+r^2) = 13 \end{cases}$

$t_1 \times t_2 \times t_3 = 27$

$\begin{cases} a \cdot ar \cdot ar^2 = 27 \\ a^3 r^3 = 27 \rightarrow ar = 3 \rightarrow a = \dfrac{3}{r} \end{cases}$

$\dfrac{3}{r}(1+r+r^2) = 13 \rightarrow 3(1+r+r^2) = 13r \rightarrow 3r^2 - 10r + 3 = 0$

$\Delta = 100 - 36 = 64$

$r = \dfrac{10 \pm 8}{6} \rightarrow r = 3 \ / \ r = \dfrac{1}{3}$

ادامه سوال ۷

$r=3 \Rightarrow a_1=1 \quad a_2=3 \quad a_3=9$

$r=\frac{1}{3} \Rightarrow a_1=9 \quad a_2=3 \quad a_3=1$

$1\times3\times9=27 \qquad 1+3+9=13$

جملات اول تا سوم ← $1, 9, 3$ ✓

سوال ۸)

$a_n = 2, 6, 18, \ldots \qquad a_n = 2\times3^{n-1}$

$\times3 \quad \times3$

الف)

$S_n = a\left\{\frac{q^n-1}{q-1}\right\} \Rightarrow 2\left\{\frac{3^{10}-1}{3-1}\right\} = \frac{2}{2}\times\frac{3^{10}-1}{2}$

$3^{10}-1$ ✓

ب)

$P_n = a_1^n \times r^{\frac{n(n-1)}{2}} \rightarrow 2^{10}\times3^{45}$ ✓

$a_1^{10}\left(r^{1+2+3+4+5+6+7+8+9}\right) \qquad r^{45}$

سوال ۹)

$(a_1q-a_1), (a_1q^2-a_1q), (aq^3-aq^2), \ldots$

$a_1(q-1), aq(q-1), (q-1)aq^2, \ldots$

$t_1 \qquad q_2$

$\frac{t_2}{t_1} = \frac{a_1q(q-1)}{a_1(q-1)} = q \qquad \frac{t_3}{t_2} = \frac{a_1q^2(q-1)}{a_1q(q-1)} = q$

$q=d$ ✓

/ /

سوال ۱۵)

الف)

$a, a+d, a+2d, \dots, a+(n-1)d$

$S_n = a + (a+d) + (a+2d) + \dots + [a+(n-1)d]$
$S_n = [a+(n-1)d] + [a+(n-2)d] + \dots + a$ } +

$2S_n = [a + [a+(n-1)d]] + [(a+d) + [a+(n-2)d]] + \dots$
$+ [a+(n-1)d + a]$

$a + [a+(n-1)d] = 2a + (n-1)d$
$(a+d) + [a+(n-2)d] = 2a + (n-1)d$
⋮
$[a+(n-1)d] + a = 2a + (n-1)d$ → n تا داریم

$2S_n = n \times ([2a + (n-1)d])$
$S_n = \frac{n}{2}[2a + (n-1)d]$ ✓

ب)

$a_1 + a_1 r + a_1 r^2 + a_1 r^3 + \dots + a r^{n-1}$

$S_n = a_1 + a_1 r + a_1 r^2 + a_1 r^3 + a_1 r^4 + \dots + a_1 q^{n-1}$
$S_n r = a_1 r + a_1 r^2 + a_1 r^3 + a_1 r^4 + \dots + a_1 r^n$

$S_n - S_n r = a_1 - a_1 r^n \rightarrow S_n(1-r) = a_1(1-q^n)$
$\Rightarrow S_n = a_1 \left[\frac{(1-q^n)}{1-r}\right]$ ✓