دهم دختر B

الیا یادگاری

۱) الف) $\frac{1}{2}, 1, 2, \ldots$ ($\times 2$, $\times 2$) $\quad a_n = \frac{1}{2}(2)^{n-1} \quad a_{10} = \frac{1}{2}(2)^9 = 2^8 = 256$

ب) $\frac{a_{17}}{a_{13}} = \frac{a_1 q^{16}}{a_1 q^{12}} = q^4 = 2^4 = 16$

ج) $\underbrace{(a_1 q^3)}_{4 = \frac{1}{2} \times 2^3} \underbrace{(a_1 q^9)}_{\frac{1}{2} \times 2^9} = x^2 \longrightarrow x^2 = 2^{10} \longrightarrow x = 2^5 = 32$

د) $\frac{1}{2} \times 2^{n-1} = 128 \longrightarrow 2^{n-1} = 256 \longrightarrow 2^8 = 256 \longrightarrow n-1 = 8 \longrightarrow n = 9$

۲) $a_1 q^4 = 12 \quad a_1 q^7 = 96 \quad \frac{a_1 q^7}{a_1 q^4} = q^3 = 8 \longrightarrow q = 2$

$a_1(2^4) = 12$

$a_1 = \frac{12}{16} = \frac{3}{4} \quad a_n = \frac{3}{4}(2)^{n-1} \longrightarrow a_{10} = \frac{3}{4}(2)^9 = 3 \times 2^7 = 384$

۳) الف) $\frac{a_3}{q^2} \times \frac{a_3}{q} \times a_3 \times a_3 q \times a_3 q^2 = a_3^5 = 243 \longrightarrow a_3 = 3$

ب) $a_1 \times a_5 = \frac{a_3}{q^2} \times a_3 q^2 = a_3^2 = 3^2 = 9$

۴) $\frac{4\sqrt{2}}{2^a} = \frac{2^b}{4\sqrt{2}} \longrightarrow 2^{a+b} = (4\sqrt{2})^2 = 32 = 2^5 \quad \frac{a+b}{2} = \frac{5}{2} = 2/5$

۵) $\frac{a_7}{a_3} = \frac{x}{1-x} = q^4 \quad \frac{a_{11}}{a_7} = \frac{x+1}{x} = q^4 \longrightarrow \frac{x}{1-x} = \frac{x+1}{x}$

$x^2 = \underbrace{(1-x)(x+1)}_{1-x^2} \longrightarrow 2x^2 = 1 \longrightarrow x^2 = \frac{1}{2} \longrightarrow x = \pm\frac{1}{\sqrt{2}} \xrightarrow{\text{گویا}} x = \pm\frac{\sqrt{2}}{2}$

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۶)

$$\overbrace{a_1(1+r^3)}^{}\quad a_1 + a_1 r^3 = 28 \qquad \underbrace{a_1 r + a_1 r^2}_{a_1 r(1+r)} = 12 \longrightarrow \frac{a_1(1+r^3)}{a_1 r(1+r)} = \frac{28}{12} = \frac{7}{3}$$

$$\frac{(1+r)(1^2 - r + r^2)}{r(1+r)} = \frac{7}{3} \longrightarrow 3(1-r+r^2) = 7r \longrightarrow 3r^2 - 10r + 3 = 0$$

$$r = \frac{-b \pm \sqrt{\Delta}}{2a} = \frac{10 \pm \sqrt{64}}{6} = \frac{10 \pm 8}{6}$$

$$r = 3 \rightarrow a_1 = 1 \rightarrow 1, 3, 9, 27$$

$$r = \frac{1}{3} \rightarrow a_1 = 27 \rightarrow 27, 9, 3, 1$$

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۷)

$$a_1 + a_1 q + a_1 q^2 = 13 \rightarrow \frac{3}{q} + 3 + 3q = 3q + \frac{3}{q} = 10 \rightarrow q = 3$$

$$a_1 \times a_1 q \times a_1 q^2 = 27 \rightarrow a_1^3 q^3 = 27 \rightarrow (a_1 q)^3 = 27 \quad a_1 q = 3$$ جمله دوم

دنباله $\rightarrow 1, 3, 9, \ldots$

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۸) الف) $S_{10} = a_1\left(\frac{q^{10}-1}{q-1}\right) \rightarrow 2\left(\frac{3^{10}-1}{3-1}\right) = 3^{10} - 1 \rightarrow \boxed{59048}$

ب) $a_n = 2(3)^{n-1} \longrightarrow a_{10} = 2 \times (3)^9$

حاصل ضرب ده جمله اول $\rightarrow (a_1 \times a_{10})^{\frac{n}{2}} \rightarrow 5 \qquad (2 \times 2 \times 3^9)^5 = (2^2 \times 3^9)^5 = \boxed{2^{10} \times 3^{45}}$

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۹)

$$a_1,\ a_1 q,\ a_1 q^2,\ a_1 q^3,\ a_1 q^4, \ldots$$

$$\overbrace{a_1 q - a_1}^{a_2 - a_1},\ \overbrace{a_1 q^2 - a_1 q}^{a_3 - a_2},\ \overbrace{a_1 q^3 - a_1 q^2}^{a_4 - a_3},\ \overbrace{a_1 q^4 - a_1 q^3}^{a_5 - a_4}$$

$$a_1(q-1) \xrightarrow{\times q} a_1 q(q-1) \xrightarrow{\times q} a_1 q^2(q-1) \xrightarrow{\times q} a_1 q^3(q-1)$$

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۱۰) الف) $a_n = a_1 + (n-1)d \qquad a_1 + (a_1+d) + (a_1+2d) + (a_1+3d) + \cdots + (a_1+(n-1)d)$

$$\frac{(n-1)(n)}{2} \longrightarrow n a_1 + \frac{n^2-n}{2} d \longrightarrow \frac{n}{2}(2a_1 + (n-1)d)$$

ب) $S_n = a_1 + a_1 q + a_1 q^2 + \cdots a_1 q^{n-1} \longrightarrow q S_n = aq + aq^2 + \cdots + aq^{n-1} + aq^n$

$$q S_n - S_n = (q-1) S_n = aq^n - a \rightarrow a(q^n - 1) \longrightarrow S_n = \frac{a(q^n-1)}{q-1}$$