Subject:
Year: Month: Day: page: ( )

سوال ششم

$\left.\begin{array}{l} a_1 + a_4 = 28 \\ a_2 + a_3 = 12 \end{array}\right\} \rightarrow \frac{a_1 + a_4}{a_2 + a_3} = \frac{28}{12} \longrightarrow \frac{a + aq^3}{aq + aq^2} = \frac{7}{3}$ $\quad \frac{a(1+q^3)}{aq(1+q)} = \frac{7}{3}$ $\quad (1+q)(q^2-q+1)$

۲

$3q^2 - 10q + 3 = 0 \Leftarrow 7q = 3q^2 - 3q + 3 \Leftarrow \frac{q^2 - q + 1}{q} = \frac{7}{3}$

$q^2 - \frac{10}{3}q + 1 = 0 \longrightarrow (q - \frac{1}{3})(q - 3) = \begin{cases} q = 3 \\ q = \frac{1}{3} \end{cases}$

استدلال درست باشد

$a_1 + a_4 = 28 \rightarrow a_1 + aq^3 = 28 \rightarrow q \begin{cases} 3 \Rightarrow a + 27a_1 = 28 \rightarrow a_1 = 1 \quad a_4 = 27 \\ \frac{1}{3} \Rightarrow a_1 + \frac{a_1}{27} = 28 \rightarrow a_1 = 27 \quad a_4 = 1 \end{cases}$

$a_2 + a_3 = 12 \Rightarrow aq + aq^2 = 12$

$\left.\begin{array}{l} q = 3 \\ q = \frac{1}{3} \end{array}\right]$ q قدر مطلق $\Rightarrow$ $q = 3 \;,\; a_1 = 1 \rightarrow a_2 = 3 ,\; a_3 = 9$ / $q = 3 \rightarrow 1, 3, 9, 27$ (a_4 بزرگترین)

$q = \frac{1}{3} \;,\; a_1 = 27 \rightarrow a_2 = 9 ,\; a_3 = 3$ / $q = \frac{1}{3} \Rightarrow 27, 9, 3, 1$ (بزرگترین a_1) عکس یکدیگر

سوال هفتم

$a_1 + a_2 + a_3 = 13 \Rightarrow \left(\frac{3}{q} + 3q = 10\right) \times q \quad 3q^2 - 10q + 3 = 0$

$a_1 \times a_2 \times a_3 = 27 \quad (a_2)^3 = 27 \quad a_2 = 3$ $\quad q_2 = 1, 3, 9$ ۲

$\frac{a_2}{a_1} \quad a_2 q$

راه حل دوم (روش دیگر) $\longrightarrow q^2 - 10q + 9 = 0 \rightarrow (q-1)(q-9) \quad \begin{cases} q = 3 \\ q = \frac{1}{3} \end{cases} \quad q_{\frac{1}{3}} = 9, 3, 1$

سوال هشتم

$A = a_1, a_2, a_3, a_4, \dots, a_n$ $\quad a_2 - a_1 = aq - a = a(q-1)$

$a, aq, aq^2, aq^3, \dots, aq^{n-1}(aq^n)$ $\quad a_3 - a_2 = aq^2 - aq = aq(q-1)$

$a_4 - a_3 = aq^3 - q^2 = aq^2(q-1)$ ۲

$A = \underbrace{a(q-1)}_{a_1}, \underbrace{aq(q-1)}_{a_2}, \underbrace{aq^2(q-1)}_{3}, \dots$ $\quad q = \frac{a_2}{a_1} = \frac{aq(q-1)}{a(q-1)} = q$

سوال نهم الف

$S_n = \frac{n}{2}(2a + (n-1)d)$

$S_n = a_1 + (a_1 + d) + (a_1 + 2d) + \dots + (a_1 + d(n-1))$ اثبات تشریحی

$S_n = \underbrace{a_n}_{a_n} + \underbrace{a_n - d}_{a_{n-1}} + \underbrace{a_n - 2d}_{a_{n-2}} + \dots + \underbrace{(a_1 + d)}_{a_2} + \underbrace{a_1}_{a_1}$ ۲

$a_1 + d(n-1) + a + d(n-2) + \dots + a_1 + d + a = \quad 2S_n = \underbrace{2a_1 + (n-1)d}_{\text{جمع اول}} + \underbrace{2a_1 + (n-1)d}_{\text{دوم}} + \dots$

$2S_n = n(2a_1 + d(n-1)) \longrightarrow S_n = \frac{n}{2}(2a_1 + d(n-1))$

ب

$S_n = a_1 + a_2 + a_3 + \dots + a_n$

$-S_n = -a_1 - aq - aq^2 - aq^3 - \dots - a_1 q^{n-1}$

Arman

$qS_n = aq + aq^2 + aq^3 + \dots + a_1 q^n$

$qS_n - S_n = aq^n - a \rightarrow S_n = a\left(\frac{q^n - 1}{q - 1}\right)$

سوال اول (۲)

الف) $a_n = \frac{1}{2}, 1, 2, \dots \to a_n = aq^{n-1}$ ؛ $a_n = \frac{1}{2}\times 2^{n-1} \to a_{10} = \frac{1}{2}\times 2^{9} = 2^{8} = 256$

ب) $a_7 = \frac{1}{2}\times 2^{n-1} \to a_7 = \frac{1}{2}\times 2^{6} = 2^{5}$ ؛ $a_{12} = \frac{1}{2}\times 2^{n-1} \to a_{12} = \frac{1}{2}\times 2^{11} = 2^{10}$ ؛ $\frac{2^{10}}{2^{5}} = 2^{5} = 32$

ج) $b^2 = ac \to b^2 = a_1 q^2 \times a_1 q^8 \to \sqrt{b^2} = \sqrt{2^{10}} \to b = 2^{5} = 32$

د) $128 = \frac{1}{2}\times q^{n-1} \to 256 = q^{n-1} \to 2^{8} = q^{n-1} \to 8 = n-1 \to n = 9$

سوال دوم (۲)

$a_5 = 12,\ a_8 = 96$: $\frac{a_8}{a_5} = q^{8-5} \Rightarrow \frac{96}{12} = q^3$ ، $q^3 = 2^3$ ، $q = 2$

$a_{10} = aq^9 \to a_{10} = \frac{3}{4}\times 2^{9} = 3\times 2^{7} = 384$ ← $a_5 = a\times 2^4 = 12 \to a = \frac{3}{4}$

سوال سوم (۲)

الف) $a_1 \times a_2 \times a_3 \times a_4 \times a_5 = 243 \to (a_3)^5 = 243 \to a_3 = 3$

ب) $a_1 \times a_5 = (a_3)^2 = 9$

سوال چهارم

$(4\sqrt{2})^2 = 2^a \times 2^b$ و $b^2 = ac$ ؛ $32 = 2^{a+b}$ و $5 = a+b$ و $\frac{a+b}{2} = \frac{5}{2}$

سوال پنجم (۱٫۷۵)

a_3, a_7, a_{11} ؛ $b^2 = ac$ ؛ $x^2 = 1 - x^2 \to 2x^2 = 1$

$x-1,\ x,\ x+1$ ؛ $x^2 = (1-x)(1+x)$

$x = \frac{4}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{4\sqrt{2}}{2}$ ؛ $x = \pm\sqrt{\frac{1}{2}}$ ، $\sqrt{x^2} = \sqrt{\frac{1}{2}}$

$\frac{-\frac{\sqrt{2}}{2}}{1+\frac{\sqrt{2}}{2}} = q^4$ ✗

اگر $x = -\frac{\sqrt{2}}{2}$ باشد q^4 منفی می‌شود و غیر قابل قبول است

سوال ششم (۲)

ب) $a_n = 2, 6, 18, \dots$ ؛ $S_{10} = 2\times\left(\frac{3^{10}-1}{3-1}\right) = 3^{10} - 1 = 59048$

الف) $a_1 \times a_2 \times a_3 \times \dots \times a_{10} = a \times aq \times aq^2 \times \dots \times aq^9 = (a_1)^{10} \times q^{45} = 2^{10}\times 3^{45}$