

الف) $a_n = \frac{1}{2^n} \rightarrow 2$ $a_n = \frac{1}{2}(2)^{n-1}$ $a_{11} = \frac{1}{2}(2)^9 = 2^8 = 256$
 ب) $a_{14} = \frac{1}{2}(2^{14}) = 2^{13}$ $a_{18} = \frac{1}{2}(a^{18}) = a^{17} \rightarrow \frac{a_{14}}{a_{18}} = \frac{2^{13}}{2^{17}} = \frac{1}{2^4} = \frac{1}{16}$
 ج) $a_8 = \frac{1}{2}(2)^8 = 2^7$ $a_{11} = \frac{1}{2}(2)^9 = 2^8$ $a_{10} = \sqrt{a_8 \times a_{12}} = \sqrt{2^7 \times 2^9} = \sqrt{2^{16}} = 2^8 = 256$
 د) $\frac{1}{2}(2^{n-1}) = 128 \rightarrow 2^{n-1} = 256 \rightarrow 2^{n-1} = 2^8 \rightarrow n-1 = 8 \rightarrow n = 9$

$a_{10} = 12$
 $a_1 = 94$ $\rightarrow d = \frac{94}{12} = 7.83$ $\rightarrow d = 8$
 $a_{10} = 9 \rightarrow a_{10} \times q^9 = 12 \times (2)^9 = 12 \times 512 = 6144$

الف) $a_1 \times a_2 \times a_3 \times a_4 \times a_5 = 243$
 $a_1 \times a_2 \times a_3 \times a_4 \times a_5 = 243 \rightarrow a_1 \times a_5 = 243$ $\rightarrow (a_1 \times a_5)^2 = 243^2 \rightarrow a_1 \times a_5 = 243$
 $a_2 = a_1 \times q$
 ب) $a_1 \times a_5 = a_1 \times a_1 \times q^4 \rightarrow a_1^2 \times q^4 = (a_1 \times q^2)^2 = (a_3)^2 = 9$

$2^a, \sqrt[4]{2}, 2^b \rightarrow (\sqrt[4]{2})^2 = (2^a \times 2^b) \Rightarrow 2^{\frac{1}{2}} = 2^{a+b} \Rightarrow 2^{\frac{1}{2}} = 2^1 \Rightarrow a+b = \frac{1}{2}$
 $\frac{a+b}{2} = \frac{a}{2}$

الف) $a_n = 1 - x$
 ب) $a_n = x$
 ج) $a_{n+1} = x+1$
 $(x)^2 = (1-x)(x+1)$
 $x^2 = 1 - x^2 \rightarrow 2x^2 = 1 \rightarrow x = \pm \frac{1}{\sqrt{2}}$
 $x^2 = 1 \rightarrow x^2 = \frac{1}{2} \rightarrow x = \pm \frac{\sqrt{2}}{2}$
 به ازای $x = -\frac{\sqrt{2}}{2}$ حاصل $\frac{a_{11}}{a_{17}} = q^6$ عددی متناهی می شود
 در صورتی که q مقبلاً نامتناهی است پس $x = -\frac{\sqrt{2}}{2}$ غیر قابل قبول است

$$\begin{cases} a_1 + a_2 = 18 \rightarrow a + aq^2 = 18 \\ a_2 + a_3 = 12 \rightarrow aq + aq^2 = 12 \end{cases} \rightarrow \frac{a(1+q^2)}{aq(1+q)} = \frac{18}{12} \rightarrow \frac{1+q^2}{q(1+q)} = \frac{3}{2}$$

$$\rightarrow \frac{(1+q)(q^2 - q + 1)}{q(1+q)} = \frac{3}{2} \rightarrow \frac{q^2 - q + 1}{q} = \frac{3}{2} \xrightarrow{\times 2q} 2q^2 - 2q + 2 = 3q$$

$$\rightarrow 2q^2 - 3q + 2 = 0 \rightarrow \Delta = 1 - 4 = -3 \rightarrow \frac{3 \pm \sqrt{-3}}{4} \quad \begin{cases} q = \frac{3}{2} \rightarrow 1, 3, 9, 27 \\ q = \frac{1}{2} \rightarrow 27, 9, 3, 1 \end{cases}$$

$$\begin{cases} a + aq + aq^2 = 18 \rightarrow a + 3 + \frac{9}{a} = 18 \rightarrow \frac{a^2 + 3a + 9}{a} = 18 \\ (a)(aq)(aq^2) = 12 \rightarrow a^3 q^3 = 12 \rightarrow aq = 2 \rightarrow q = \frac{2}{a} \end{cases}$$

$$a^2 + 3a + 9 = 18a \rightarrow a^2 - 15a + 9 = 0 \rightarrow (a-1)(a-9) = 0 \quad \begin{cases} a=1 \rightarrow 1, 3, 9 \\ a=9 \rightarrow 9, 3, 1 \end{cases}$$

$$a_n = 2, 9, 18 \rightarrow a_n = 2(3)^{n-1}$$

$$S_1 = 2 \left(\frac{3^n - 1}{3 - 1} \right) = 3^n - 1 = 27 - 1 = 26$$

$$P_1 = \sqrt{(a_1 \times a_{10})^9} = \sqrt{(2 \times 18)^9} = \sqrt{2^9 \times 3^{18}} = 2^4 \times 3^9 = 16 \times 19683 = 314928$$

$$a_n = a, aq, aq^2, \dots$$

$$b_n = aq^n, aq^{n-1} \Rightarrow aq^{n-1}(q-1)$$

$$b_1 = a(q-1), b_2 = aq(q-1), b_3 = aq^2(q-1)$$

$$\frac{b_r}{b_1} = \frac{aq^r(q-1)}{a(q-1)} = q^r \rightarrow \text{قد نستخدمه}$$

$$a + (a+d) + (a+2d) + \dots + (a+(n-1)d) \rightarrow a + (a+d) + (a+2d) + \dots + (a+(n-1)d)$$

$$[a + (n-1)d] + [a + (n-2)d] + \dots + a \rightarrow a + a + (n-1)d = 2a + (n-1)d$$

$$\rightarrow S_n + S_n = 2S_n = n(2a + (n-1)d) \Rightarrow \frac{2}{2} n(2a + (n-1)d) = n(2a + (n-1)d)$$

$$S_n = a + aq + aq^2 + \dots + aq^{n-1} \quad \times q \quad S_n q = aq + aq^2 + \dots + aq^{n-1} + aq^n \quad \frac{S_n q - S_n}{q-1} = aq^n - a$$

$$\rightarrow (q-1)S_n = a(q^n - 1) \div q-1 \quad S_n = \frac{a(q^n - 1)}{q-1}$$