۱)

$a_n = aq^{n-1} \to q=2 \qquad a=\frac{1}{2} \to \frac{2^{n-1}}{2} \to 2^{n-2}$

۲۰ عالیه :))

۲

الف) $\frac{2^{10-1}}{2} \to \frac{2^9}{2} = 2^8$

ب) $\frac{a_{17}}{a_{13}} = q^{17-13} \to q^4 \to 2^4 = 16$

ج) $q^{n-1} = \frac{b}{a} \to \frac{aq^9}{aq^4} = q^{n-1} \to q^{n-1} = q^4 \quad a = n-1 \to n=7$

$a_7 = 2^{7-2} \to 2^5 = 32$

د) $2^{n-2} = 128 \to 2^{n-2} = 2^7 \to n=9$

۲)

$a_5 = 12 \to aq^4$

$a_8 = 96 \to aq^7$

$\frac{a_8}{a_5} = q^{8-5} \to q^3 = \frac{96}{12} \to q^3 = 8 \to q=2$

$aq^4 = 12 \to a \times 2^4 = 12 \to 16a = 12 \to a = \frac{3}{4}$

$a_{10} = aq^9 \to \frac{3}{4} \times 2^9 \to 3 \times 2^7 = 384$

۲

۳)

$\underbrace{a_1 a_2}_{a_3^2} a_3 a_4 a_5 = 243 \to a_3^2 a_3^2 a_3 \to a_3^5 = 243 \qquad \sqrt[5]{243} = 3$

$a_3 = 3$

۲

الف) $a_3 = 3$

ب) $a_1 a_5 = a_3^2 \to 3^2 = 9$

۴)

$(4\sqrt{2})^2 = 2^{a+b} \to 32 = 2^{a+b} \to 2^5 = 2^{a+b} \to a+b = 5$

واسطه حسابی $\to \frac{a+b}{2} \to \frac{5}{2}$

۲

۵)

$aq^2 = 1-x$

$aq^6 = x \longrightarrow$ واسطه هندسی a_3, a_{11}

$aq^{10} = x+1$

۲

$x^2 = (1-x)(1+x) \to 1-x^2 = x^2 \to 2x^2 = 1 \to x^2 = \frac{1}{2} \to x = \frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} \to \frac{\sqrt{2}}{2}$

۶)

$a + aq^4 = 28 \longrightarrow a(1+q^4)$

$aq + aq^3 = 12 \longrightarrow a(q^3+q)$

$\dfrac{28}{12} = \dfrac{1+q^4}{q^3+q} \longrightarrow \dfrac{1+q^4}{q(q^2+1)}$

$\longrightarrow 12 + 12q^4 = 28q + 28q^3$

$12q^4 - 28q^3 - 28q + 12 = 0 \longrightarrow 4(3q^4 - 7q^3 - 7q + 3) = 0 \longrightarrow 3q^4 - 7q^3 - 7q + 3 = 0$ ۲

$\longrightarrow a(1+q^4) = 28 \begin{cases} q=3 & a=1 \\ q=\frac{1}{3} & a=27 \end{cases}$

$1, 3, 9, 27$ $\quad$ $27, 9, 3, 1$ $\longrightarrow$ ۲۷ صحیح

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۷)

$a_1 + a_2 + a_3 = 13 \longrightarrow a + aq + aq^2$

$a_1 a_2 a_3 = 27 \longrightarrow a^3 q^3 = 27 \longrightarrow aq = \sqrt[3]{27} \longrightarrow aq = 3$ ۲

$\hookrightarrow a_1 a_3 = \dfrac{27}{3} \Rightarrow 9 = a_1 a_3$ $\quad$ a فرض کنیم

$\dfrac{3}{q} + 3q = 13 - 3 \to 10 \longrightarrow 3\left(q + \dfrac{1}{q}\right) = 10 \longrightarrow q + \dfrac{1}{q} = \dfrac{10}{3} \longrightarrow \dfrac{q^2+1}{q} = \dfrac{10}{3} \longrightarrow 3q^2 + 3 = 10q$

$\longrightarrow 3q^2 + 3 - 10q = 0 \longrightarrow q^2 - 10q + 9 = 0 \longrightarrow (q \mp 1)(q \mp 9) = 0$ $\quad \xrightarrow{\div 3} \frac{1}{3}, 3$

$9\ 3\ 1$

$1, 3, 9$

هر دو حالت درست

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۸)

$a_n = 2, 6, 18, \ldots \quad a_1 = 2 \quad q = 3$

الف) $S_{10} = a_1\left(\dfrac{q^n - 1}{q-1}\right) \longrightarrow 2\left(\dfrac{3^{10}-1}{2}\right) \Rightarrow 3^{10} - 1$ ۲

ب) $P_{10} = (\text{اول} \times \text{آخر})^{\frac{n}{2}} \longrightarrow (2 \times 3^9 \times 2)^5 \longrightarrow (4 \times 3^9)^5$

$a_{10} = 2(3)^{10-1} \longrightarrow 2 \times 3^9$

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۹)

[struck through]

$a_n = aq^{n-1}$ ۲

$d_n = a_{n+1} - a$

$d_n = aq^n - aq^{n-1} \longrightarrow a(q-1)q^{n-1}$

$\dfrac{d_{n+1}}{d_n} \longrightarrow \dfrac{a(q-1)q^n}{a(q-1)(q^{n-1})} = q$

اگر $q = 1 \longrightarrow d_n = 0$

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۱۰)

الف) $S_n = \frac{n}{2}(2a_1 + (n-1)d) \,/\, \frac{n}{2}(a_n + a_1)$ ۲

$(a) + (a+d) + (a+2d) + \cdots + (a+(n-1)d) \longrightarrow na + (d + 2d + \cdots + (n-1)d)$

$d(1 + 2 + \cdots + (n-1)) \longrightarrow d\left(\dfrac{n(n-1)}{2}\right) \Rightarrow an + d\left(\dfrac{n(n-1)}{2}\right)$

$\longrightarrow \frac{n}{2}(2a_1 + (n-1)d)$ ∎

ب) $S_n = a\left(\dfrac{q^n - 1}{q-1}\right)$

$(a + aq + aq^2 + \cdots + aq^{n-1}) \xrightarrow{\times q} aq + aq^2 + \cdots + aq^n$

$S_n - qS_n = a - aq^n \longrightarrow S_n(1-q) \longrightarrow S_n = a\left(\dfrac{1-q^n}{1-q}\right)$ ∎