

$$a_1 = 80 \Rightarrow a_5 - a_1 = 3d = 21 \Rightarrow d = 7 \Rightarrow a_n = a_1 + (n-1)d = 80 + (n-1)7$$

$$a_8 = 121 = 80 + (n-1)7 \Rightarrow (n-1)7 = 41 \Rightarrow n-1 = 22 \Rightarrow \boxed{n = 23}$$

$$a_n = 7n + 3$$

$$1 - 7n + 3 \leq 999 \Rightarrow 4 \leq 7n \Rightarrow 12 \leq n \leq 142$$

$$\frac{12+142}{2} + 1 = \boxed{129}$$

$$a_{n+1} + a_{n+2} + a_{n+3} = -4n + 12 \Rightarrow 3a_1 + d(1+n+1+n+2) = -4n + 12$$

$$3nd + 3d + 3a_1 = -4n + 12 \Rightarrow 3d = -4 \Rightarrow d = -\frac{4}{3}$$

$$-4 + 3a_1 = 12 \Rightarrow 3a_1 = 16 \Rightarrow a_1 = \frac{16}{3}$$

$$a_{10} + a_1 = 2a_1 + 2d = 12 - \frac{8}{3} = \boxed{-\frac{12}{3}}$$

$$r^n - 1, r^{n+1} - 1, r^n - r$$

$$a_r - a_n = \omega d \Rightarrow a_{1r} - a_n = a_n - a_{1r}$$

$$r^n - 1 - r^{n+1} + 1 = r^n - r \Rightarrow r^n + 1 - r^{n+1} - 1 = -r \Rightarrow r^n - r^{n+1} = -r$$

$$(r^n - 1)(r + 1) = 0 \Rightarrow r^n = 1 \Rightarrow n = 2 \Rightarrow S = 3 + 1 + 13 = \boxed{17}$$

$$a_{1r} - a_{10} = 2d = 0 \Rightarrow d = 0$$

$$a_1 + a_{1r} = 20 \Rightarrow a_1 = -\frac{20}{2} = -10$$

$$a_{1r} = 10 \Rightarrow a_{10} = 10$$

$$a_{11} = a_1 + 10d = -\frac{20}{2} + \frac{20 \times 0}{2} = \frac{0}{2} = \boxed{0}$$

$$a_1 + a_2 + a_3 + \dots + a_n = \frac{1}{p} (a_1 + a_2 + a_3 + \dots + a_n)$$

$$\omega a_1 + 1 \cdot d = \frac{1}{p} (\omega a_1 + d \cdot d)$$

$$a_1 + p d = \frac{1}{p} (a_1 + p d)$$

$$\frac{p}{p} a_1 = \frac{1}{p} d \Rightarrow p a_1 = d$$

$$\frac{a_1}{a_1} = \frac{\frac{d}{p} + d}{\frac{d}{p}} = \frac{\frac{p}{p} d}{\frac{d}{p}} \quad \boxed{p}$$

$$t_n = t_{n-1} + 1 \cdot \omega$$

$$\frac{(t_9 - t_0)}{t_9} = \frac{(t_9 - t_0) \left(\frac{t_9}{p} \right)}{t_9} = \frac{t_9}{p} = p d = p \cdot \omega = 1 \cdot \omega \quad \boxed{1 \cdot \omega}$$

$$t_9 = t_1 + 1 \cdot \omega$$

$$t_0 = t_1 + 1 \cdot \omega$$

$$t_9 = t_1 + 1 \cdot \omega$$

$$t_9 = t_1 + 1 \cdot \omega = t_1 + \omega \Rightarrow t_9 - t_1 = \omega = 1 \cdot \omega \Rightarrow d = \omega$$

$$d = \frac{t_9 - t_0}{p+1} = \frac{p + \sqrt{p} - p + 1 \cdot \sqrt{p}}{1 \cdot p} = \frac{1 \cdot \sqrt{p}}{1 \cdot p} = \boxed{\sqrt{p}}$$

$$\text{increasing} = \text{increasing} \Rightarrow a_{n+1} = 11 = a_1 + (n+1)d$$

$$d = \frac{p - p}{n - p} = \frac{p}{n - p} = \frac{10}{p - 1}$$

$$p + (n) \times \frac{10}{p-1} = 11 \Rightarrow \frac{10n}{p-1} = 1 \Rightarrow 10n - p = \omega n \Rightarrow \boxed{n=p}$$

$$a_n + a_{n+1} = a_1 + a_2 \Rightarrow \cancel{p} a_1 + \frac{(n-1+n+1)}{p+n} d = \cancel{p} a_1 + 1 \cdot d$$

$$a_n = \frac{p}{p} a_1$$

$$p+n = 1 \cdot n \Rightarrow n = 1$$

$$a_n = \frac{p}{p} a_1$$

$$a_1 + p d = p a_1 + 1 d \Rightarrow p d = -p a_1 \Rightarrow d = -a_1$$

$$a_n = a_1 + (n-1)d \Rightarrow a_1 + (n-1)(-a_1) = 0$$

$$a_1 \left(\frac{1-n+1}{p-n} \right) = 0 \Rightarrow p-n = 0 \Rightarrow \boxed{n=p}$$