

$$a_1 + 3d = 41 \rightarrow 50 + 3d = 41 \rightarrow 3d = 21 \rightarrow d = 7$$

$$a_n = 7n + 32 \quad 7n + 32 = 201 \quad 7n = 169 \quad n = 24$$

$$100 \leq 7n + 32 \leq 999$$

$$97 \leq 7n \leq 967 \rightarrow 13, \dots \leq n \leq 138, \dots \rightarrow 14 \leq n \leq 138$$

$$\frac{138 - 14}{1} + 1 = 125$$

$$a_{n+1} + a_{n+2} + a_{n+3} = -4n + 12 \quad \text{بنا بر } a_n = -2n + 1$$

$$a(n+1) + a(n+2) + a(n+3) = 2an + 4n + 3 = -4n + 12$$

$$3a = -4 \rightarrow a = -\frac{4}{3} \quad \left. \begin{array}{l} a_{14} = -2(14) + 1 = -27 \\ a_n = -2(1) + 1 = -1 \end{array} \right\} \rightarrow -36$$

$$-4(-\frac{4}{3}) + 3a = 12 \rightarrow a = 8$$

$$r^{n+1} - r^n + 1 = r^{2n} - r^{n+1}$$

$$y = r^n$$

$$ry - y + 1 = y^2 - ry \rightarrow y^2 - 2y - 1 = 0 \rightarrow (y-1)(y+1) = 0 \rightarrow y = 1 \text{ or } y = -1$$

$$a_3 + a_1 + a_{13} = y - 1 + ry + y^2 - r = 1 - 1 + 1 + 1 - 1 = 1$$

$$2d = 20 \rightarrow d = 10 \quad a_1 = a_1 + 3d \rightarrow -12d + 20 = 3 \times 10 \quad (1)$$

$$a_1 + ad + a_1 + 11d = 2a_1 + 12d = 20 \quad a_1 = -12d$$

$$\frac{a}{r} (a_1 + a_n) = \frac{1}{r} \times \frac{a}{r} \rightarrow \frac{a}{r} (a_1 + a_n) \rightarrow 2a_1 + 12d = \frac{a}{r} a_1 + \frac{12a}{r} d$$

$$\frac{a}{r} (a_1 + a_n) = \frac{a}{r} (a_1 + a_n)$$

$$2ar + 12d = \frac{1}{r} ar + 12d$$

$$\frac{1}{r} ar = 12d \rightarrow ar = 12d$$

$$\frac{ar}{a_1} = \frac{12d}{12d} = 1$$

$$t_n - t_{n-1} = r d = 1 \Rightarrow d = \frac{1}{r}$$

$$\frac{(t_n - t_0)(t_n + t_0)}{t_n} = \frac{r_0(t_n + r d + t_0 - d)}{t_n + d} = \frac{r_0(r t_n + 1)}{t_n + \frac{1}{r}} =$$

$$\frac{r_0(t_n + \frac{1}{r})}{t_n + \frac{1}{r}} = r_0 \quad \checkmark$$

$$d = \frac{r + \sqrt{r} - r + 1 \sqrt{r}}{1 \sqrt{r}} = \frac{1 \sqrt{r}}{1 \sqrt{r}} = \sqrt{r} \quad \checkmark$$

$$r_0 \quad r_1 \quad a_{n-1} = r_1 \quad a_k = a_1 + (k-1)d$$

$$a_{n+1} = 1 \rightarrow a_{n-1} - a_{n+1} = r_1 \quad a_{n-1} = r + (r_n - r)d = r_1$$

$$d = \frac{r_0}{r_n - r} \rightarrow r_n - r_1 = r a_n - r_0 \rightarrow r_n = a \rightarrow n = r \quad \checkmark$$

$$\text{write} \rightarrow (r_n - r) \left(\frac{r_0}{r_n - r} \right) = r_1 \rightarrow \frac{r_0 n - r_0}{r_n - r} = r_1 \quad \frac{r_0(r_n - r)}{r(r_n - r)} =$$

$$a_n + a_n + r = a r + a_1 r \quad a_n = r a \frac{n}{r}$$

$$a_1 + (n-1)d = r \left[a_1 + \left(\frac{n}{r} - 1 \right) d \right] \rightarrow a + (n-1)d - r a_1 - \frac{r n}{r} d + r d = 0$$

$$a_k = 0 \rightarrow a_k = a + (k-1)d \quad -r a_1 + \frac{-n + r}{r} d = 0$$

$$a_1 = -d \rightarrow -r d = r a_1 \quad (-1 + r)d = r a_1$$

$$k - r = 0 \rightarrow k = r \quad \checkmark$$