۲۰ آفرین

①

$a_1 = 40$

$a_4 = 41 \rightarrow a_1 + 3d$

$\begin{cases} a_1 = 40 \\ a_1 = 91 - 3d \end{cases} \rightarrow 3d = 91 - 40 \rightarrow 3d = 21$, $d = 7$

$208 = a_1 + (n-1)d \rightarrow 208 = 40 + 7n - 7 \rightarrow 7n = 208 - 33 \rightarrow 7n = 175 \rightarrow n = 25$

②

$\frac{100}{7} = 14$ باقی: ۲ $a_1 = 100 + 1 \Rightarrow 101$

$\frac{999}{7} = 142$ باقی: ۵ $a_n = 999 - 2 \Rightarrow 997$

$d = 7$

$997 = a_1 + (n-1)d \rightarrow 997 = 101 + 7n - 7 \rightarrow 997 = 94 + 7n \rightarrow 7n = 903$, $n = 129$

③

$a_{n+1} \rightarrow a + nd \Rightarrow$ $a_{n+1} + a_{n+2} + a_{n+3} = 3a + (3n+3)d = -9n + 12$

$a_{n+2} \rightarrow a + (n+1)d$ $\llcorner 3d = -9 \rightarrow d = -3$

$a_{n+3} \rightarrow a + (n+2)d$ $3a + 3d = 12 \rightarrow 3a = 18 \rightarrow a = 9$

$9 + (n-1)3 \rightarrow -3n + 8$

$a_{11} = 8 - 34 = -26$

$a_8 = -16 + 8 = -8$

$-26 - 8 = -34$

④

$a_4 = 2^x - 1$ $a_8 = 2^{x+1}$ $a_{12} = 2^{2x} - 3$

$2a_8 = a_4 + a_{12} \rightarrow 2(2^{x+1}) = 2^{2x} - 3 + 2^x - 1 \rightarrow 2^{2x} + 2^x - 4 = 4 \times 2^x$

$2^{2x} - 3 \times 2^x - 4 = 0 \xrightarrow{y = 2^x} y^2 - 3 \times y - 4 = 0 \rightarrow (y-4)(y+1) \rightarrow y = 4 \rightarrow 2^x = 4 \rightarrow x = 2$

$a_4 = 4 - 1 = 3$ $3 + 8 + 13 = 24$

$a_8 = 2^3 = 8$

$a_{12} = 2^4 - 3 = 13$

⑤

$a_{12} - a_{10} = 2d \rightarrow 5 \rightarrow d = 2/5$

$a_{12} + a_{10} = 25 \rightarrow 2a_1 + 20d = 25 \rightarrow 2a_1 = 25 - 50 \rightarrow a_1 = -12/5$

$a_{21} = -12/5 + 20 \times (2/5) \rightarrow 50 - 12/5 = 37/5$

$S_{20} = a_1 + a_2 + a_3 + a_4 + a_5 \longrightarrow 20a + 10d$

$a_5 + a_6 + \dots + a_{11} \longrightarrow 30a + 75d$

$3(20a + 10d) = 30a + 75d$

$60a + 30d = 30a + 75d \longrightarrow 30a - 45d = 0 \longrightarrow 30(2a - 3d) = 0 \longrightarrow 2a = 3d$

$a = \frac{3}{2}d$

$a_2 = a + d \longrightarrow a + 2a \longrightarrow 3a$

$\frac{3a}{a} = 3$ ✓

$a = a_{n-2} + 10 \longrightarrow a + (n-1)d \longrightarrow a + (n-4)d + 10 \longrightarrow (n-1)d = (n-4)d + 10 \longrightarrow (n-1)d - (n-4)d = 10$

$\longrightarrow nd - d - nd + 4d = 10 \longrightarrow d = 2$

$\hookrightarrow a_0 = a + 4d = a + 8$

$a_1 = a + 4d = a + 12$

$a_4 = a + 8d = a + 16$

$a_4^2 - a_0^2 \longrightarrow (a_4 - a_0)(a_4 + a_0) \longrightarrow (a + 16 - a - 8)(a + 16 + a + 8)$

$\hookrightarrow 8(2a + 24) \longrightarrow 16a + 192 = a_4^2 + a_0^2$

$\frac{a_4^2 - a_0^2}{a_1} = \frac{16a + 192}{a + 12} = 16$ ✓

$a = 2(1 - 4\sqrt{2}) \longrightarrow 2 - 8\sqrt{2}$

$b = 2 + \sqrt{2}$

$d = \frac{b - a}{n - 1} \longrightarrow \frac{2 + \sqrt{2} - (2 - 8\sqrt{2})}{12 + 1} \longrightarrow \frac{2 + \sqrt{2} - 2 + 8\sqrt{2}}{12} = \frac{12\sqrt{2}}{12} \Rightarrow \sqrt{2}$ ✓

$b = 32, a = 2, n = 4n - 3$

$\frac{32 - 2}{4n - 3 + 1} = \frac{16}{4n - 2} \longrightarrow \frac{15}{2n - 1} \longrightarrow d$

$a_{n+1} \longrightarrow a_1 + nd \longrightarrow 2 + n\left(\frac{15}{2n-1}\right) = 11 \longrightarrow n\left(\frac{15}{2n-1}\right) = 9 \longrightarrow 15n = 18n - 9 \longrightarrow n = 3$

$\frac{15}{9 - 1} = \frac{d}{2}$ ✓

$2, 5, 8, \boxed{11}$ پاسخ

$\begin{cases} a_n + a_{n+4} \longrightarrow 2a_{n+2} \\ a_2 + a_6 \longrightarrow 2a_4 \end{cases} \longrightarrow 2a_{n+2} = 2a_4 + 16d \longrightarrow 2a_n + 4d = 2a_4 + 16d \longrightarrow 2a_n = 2a_4 + 12d \longrightarrow a_n = a_4 + 6d \longrightarrow n = 8$

$a_1 = 2a_4 \longrightarrow a + 7d = 2a_1 + 6d \longrightarrow 7a + 7d = 0 \longrightarrow \underbrace{a + d}_{a_2} = 0$

$a_2 = 0$ ✓