

(1)

$$\begin{aligned} (1)^2 + 0 & \quad (2)^2 + 1 & (3)^2 + 3 & (4)^2 + 6 \\ n^2 + \frac{n(n-1)}{2} &= \frac{1}{2}n^2 - \frac{1}{2}n \rightarrow a_n = \frac{1}{2}n^2 - \frac{1}{2}n \\ n=10 &\rightarrow \frac{1}{2} \times 100 - 5 = \boxed{45} \end{aligned}$$

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$$a_n = \infty \rightarrow n = ?$$

$$n = 0, 1, 3, \dots$$

$$\begin{aligned} \frac{n(n-1)}{2} & \quad \frac{n(n-1)}{2} = 55 \rightarrow n^2 - n - 110 = 0 \\ \boxed{n=11} & \quad (n-11)(n+10) = 0 \end{aligned}$$

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$$\begin{aligned} 5 + 0 \times 4 & \quad 9 + 1 \times 4 = 13 & 12 + 2 \times 4 = 20 \rightarrow b_n = a_n + 4(n-1) = 8n - 4 \\ 5 & \quad 9 & 13 \rightarrow a_n = 4n + 1 \\ 1 + 4 \times 1 & \quad 1 + 4 \times 2 & 1 + 4 \times 3 \\ n=11 & \rightarrow \begin{cases} \text{رنگی} = 4 \times 11 + 1 = 45 \\ \text{کل} = 45 + 8 \times 10 = 125 \end{cases} \\ & \quad \text{کل} = (4 \times 11 + 1) + 4(10) = 45 + 40 = 85 \end{aligned}$$

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$$a_n = \frac{(-1)^n}{n} \quad \begin{aligned} n=1 &\rightarrow -1 & n=3 &\rightarrow -\frac{1}{3} \\ n=2 &\rightarrow \frac{1}{2} & n=4 &\rightarrow \frac{1}{4} \end{aligned}$$

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همه n بی‌نهایتی a_n فرد باشد از a_n به صورت زیر می‌آید: a_n از $\frac{1}{n}$ به سمت صفر میل می‌کند

$$a_{min} = -1, \quad a_{max} = \frac{1}{2} \quad \text{تفاوت} = \frac{1}{2} - (-1) = \boxed{\frac{3}{2}}$$

$$b_n = (-3)^n - v_n, \quad n=3 \rightarrow -27 - 21 = -48 = b_3$$

$$a_n = -5n + 22 \rightarrow n=k \Rightarrow a_k = -5k + 22$$

$$a_k = b_k \Rightarrow -5k + 22 = -48 \Rightarrow 5k = 70 \Rightarrow \boxed{k=14}$$

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$a_{10} - a_9 = 12 \Rightarrow a_{10} = a_9 + 12d \Rightarrow a_9 + 12d - a_9 = 12 \Rightarrow 12d = 12 \Rightarrow d = 1$
 $a_9 - a_1 = ? \Rightarrow a_9 = a_1 + 8d \Rightarrow a_1 + 8d - a_1 = 8d \Rightarrow 8 \times 1 = 8$
 $a_9 - a_{10} = 12 \quad -12d = 12 \rightarrow d = -1$
 $a_1 - a_9 = a_1 - a_1 - 8d \Rightarrow -8d = 8 \Rightarrow d = -1$

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$a_7 = 10 \Rightarrow a_7 - a_6 = 12d = 12 - 10 = 2 \Rightarrow d = \frac{1}{6}$
 $a_7 = 10 = a_6 + d$
 $a_1 = b_1 = 10$
 $a_7 = b_7 = a_1 + 6d = 10 + 1 = 11$
 $d = b_7 - b_1 = 11 - 10 = 1$

$a_n = a_1 + (n-1)d = 10 + (n-1) \times \frac{1}{6} = \frac{6n+1}{6}$
 $b_n = 10 + (n-1) \times 1 = n+9$

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$t_n = \frac{1}{8}, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1, \frac{5}{4}, \dots$
 $t_n = \frac{5n-3}{4n+3}$
 $\frac{5n-3}{4n+3} < \frac{1}{2} \Rightarrow 5n-3 < 2n+3 \Rightarrow 3n < 6 \Rightarrow n < 2$
 $n < 2 \Rightarrow n = 1$

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$t_n = 4, 9, 16, 25, \dots$
 $t_n = An^2 + Bn$
 $4A = -3 \Rightarrow A = -\frac{3}{4}$
 $n=1 \Rightarrow -\frac{3}{4} + B = 4 \Rightarrow B = \frac{19}{4}$
 $3A + 2B = -\frac{9}{4} + \frac{38}{4} = \frac{29}{4} = 7.25$

7.25

$t_n = -4, -8, -12, \dots$
 $t_n = n^2 - n$
 $a_7 = 11$
 $a_7 = 49 = a_1 + 6d \Rightarrow a_1 + 6d = 49$
 $a_n = n^2 - n$
 $n^2 - n = 49 \Rightarrow n^2 - n - 49 = 0$
 $(n-9)(n+5) = 0 \Rightarrow n = 9$
 $t_9 = a_9 = 81 - 9 = 72$

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