کیمیا مرادی - تکلیف ۹ ۱۸/۱۷/۵

1- ۲

الف) $y = x^2 - 5$

$x^2 = y + 5 \rightarrow x = \pm\sqrt{y+5} \rightarrow y+5 \geq 0 \rightarrow y \geq -5$

$[-5, +\infty)$

ب) $y = x^3 + 1$

$x^3 = y - 1 \rightarrow x = \sqrt[3]{y-1} \rightarrow R_f = \mathbb{R}$

2- ۲

الف) $y = x^2 - 4x + 6 \rightarrow$

$x^2 - 4x + 6$

$(x-2)^2 + 2 = y$

$(x-2)^2 = y - 2 \rightarrow x - 2 = \pm\sqrt{y-2}$

$x = \pm\sqrt{y-2} + 2 \rightarrow R_f = [2, +\infty)$

$y \geq 2$

ب) $y = x^2 - \frac{5x}{2} + 1 + \frac{25}{16} - \frac{25}{16}$

$= \left(x^2 - \frac{5}{2}x + \frac{25}{16}\right) \rightarrow \left(x - \frac{5}{4}\right)^2$

$y = \left(x - \frac{5}{4}\right)^2 + \underbrace{1 - \frac{25}{16}}_{-\frac{21}{16}}$

$y - \frac{21}{16} = \left(x - \frac{5}{4}\right)^2$

$\sqrt{y - \frac{21}{16}} = x - \frac{5}{4} \rightarrow \sqrt{y - \frac{21}{16}} - \frac{5}{4} = x$

$y \geq \frac{21}{16}$

$[\frac{21}{16}, +\infty)$

3- ۱/۵

الف) $y = \frac{x^2 + 3}{x^2 - 2} \rightarrow yx^2 - 2y = x^2 + 3$

$yx^2 - x^2 = 2y + 3$

$x = \pm\sqrt{\frac{2y+3}{y-1}} \leftarrow x^2 = \frac{2y+3}{y-1} \leftarrow x^2(y-1) = 2y+3$

$y - 1 \geq 0 \rightarrow y > 1 \quad (-\infty, -\frac{3}{2}] \cup (1, +\infty)$

$-\frac{3}{2}$, 1: $+$ | $-$ | $+$

ب) $y = \frac{2|x|+1}{|x|-4} \to y|x| - 4y = 2|x|+1$

$y|x| - 2|x| = 4y+1$

$|x|(y-2) = 4y+1 \to |x| = \frac{4y+1}{y-2}$

$-\frac{1}{4}$		2	
$+$	$-$		$+$

$(2, +\infty) \cup (-\infty, -\frac{1}{4}]$

4 (4)

$y = \frac{1}{x^2 - 4x}$

$yx^2 - 4xy = 1$

$yx^2 - 4xy - 1 = 0 \to \Delta = 16y^2 + 4y$

$\Delta = \frac{4y \pm \sqrt{16y^2 + 4y}}{2y}$

$\sqrt{16y^2 + 4y} \geq 0 \to 4y(4y+1) \geq 0$

$-\frac{1}{4}$		0	
$+$	$-$		$+$

$R_f = \{(-\infty, -\frac{1}{4}] \cup [0, +\infty)\} - \{0\}$

5 (3)

الف) $y = x^2 - 6x + 2 \to \min \to \begin{bmatrix} 3 \\ -7 \end{bmatrix}$

$[-7, +\infty)$

ب) $-x^2 + 4x + 2 \to \max \to \begin{bmatrix} 2 \\ 6 \end{bmatrix}$

$\frac{-\Delta}{4a} \to 16 - (4 \times -1 \times 2) + 8 = \frac{-24}{-4} = 6$

$R_f = (-\infty, 6]$

-۶

الف) $y = \sqrt{x^2 - 6x + 2}$

$\min \begin{cases} x = \frac{6}{2} = 3 \\ y = \frac{-\Delta}{4a} = \frac{36 - 8}{} = \frac{-28}{4} = -7 \end{cases}$ ③

$x \geq 0$ $\quad R_f = [0, +\infty)$

ب) $y = \sqrt{-x^2 + 4x + 15}$

$\max \begin{cases} x = 2 \\ y = 19 \ldots = \frac{56}{4} = 14 \end{cases}$

$\sqrt{[-\infty, +14]} \rightarrow [0, \sqrt{+14}]$

-۷

الف) $y = x^7 + 3x^4 + 2x + 1 \rightarrow$ درجه فرد → $R_f = \mathbb{R}$ ③

ب) $y = \sqrt{x^5 + 4x^4 + 6x + 1}$

$\sqrt{\mathbb{R}} = \mathbb{R}^+ = [0, +\infty)$

-۸ ③

الف) $y = \frac{3x+1}{x-2}$ $\quad \mathbb{R} - \{3\}$

ب) $y = \sqrt{\frac{4x+1}{x+3}}$ $\sqrt{\mathbb{R} - \{4\}} \rightarrow [0, +\infty) - \{2\}$

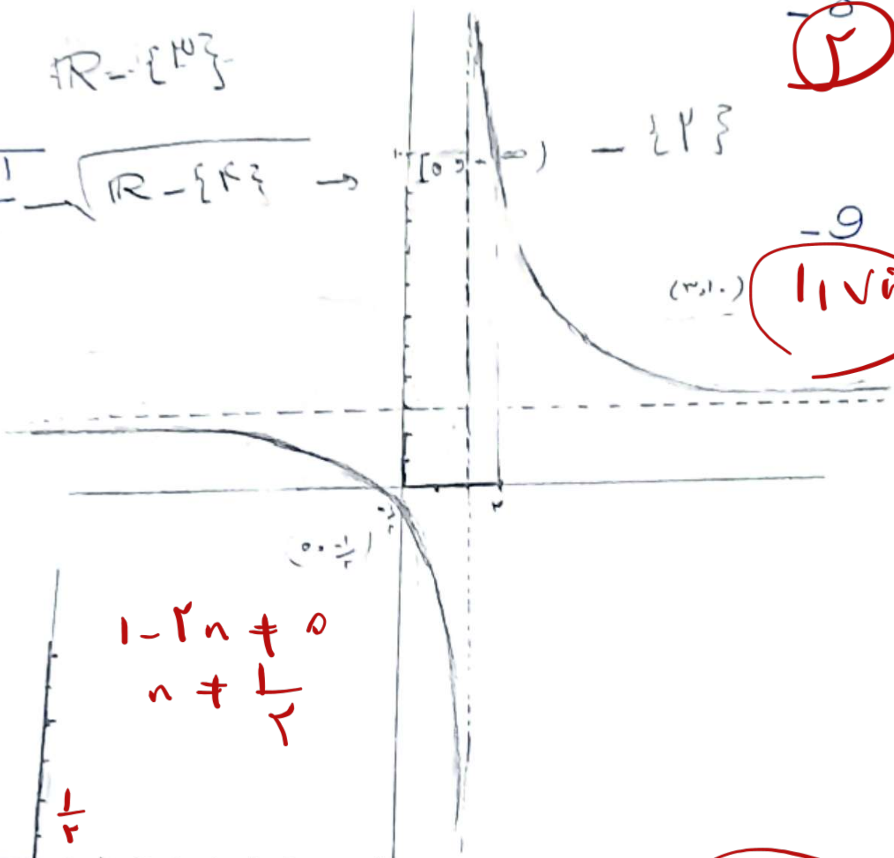
-۹ (۳،۱،۰) ۱،۱۷۵

الف) $y = \frac{3x+1}{x-2}$

مجانب قائم $= 2$

مجانب افقی $= 3$

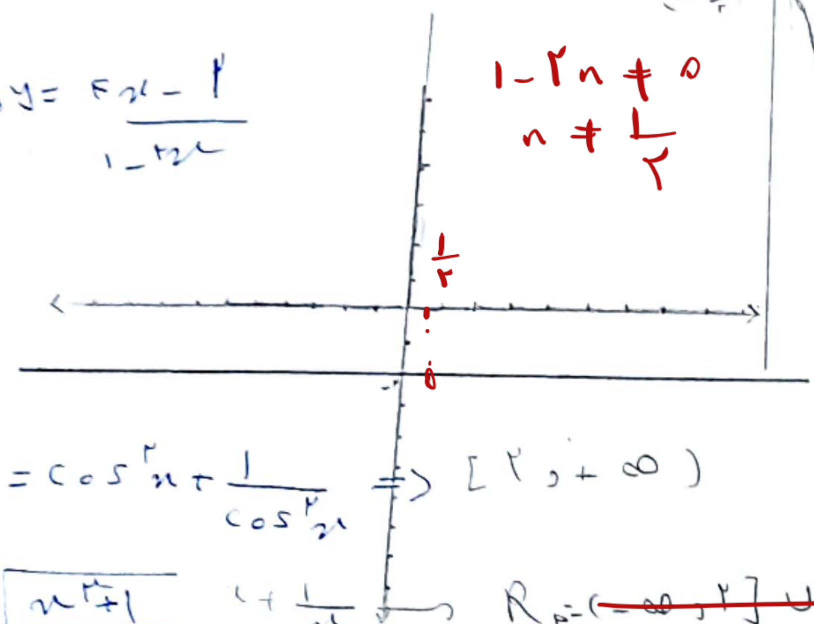
$(0, -\frac{1}{2})$



ب) $y = \frac{4x - 2}{1 - 2x}$

$1 - 2x \neq 0$

$x \neq \frac{1}{2}$



۱،۵ -۱۰

الف) $y = \cos^2 x + \frac{1}{\cos^2 x} \Rightarrow [2, +\infty)$

ب) $y = \sqrt[3]{\frac{x^3+1}{x}}$ $\quad x^2 + \frac{1}{x} \rightarrow R_f =$ ~~$(-\infty, 2] \cup [2, +\infty)$~~

$x > 0 \rightarrow (\sqrt[3]{2}, +\infty)$

$x < 0 \rightarrow (-\infty, \sqrt[3]{-2})$