

الف) $y = x^2 - 5$

$x^2 = -y - 5$
 $x = \pm \sqrt{-y - 5} \geq 0 \rightarrow y \leq -5 \rightarrow R_f = (-\infty, -5]$

ب) $y = x^3 + 1 \rightarrow x^3 = 1 - y \rightarrow x = \sqrt[3]{1 - y} \rightarrow R_f = \mathbb{R}$

الف) $y = x^2 - 4x + 9 \rightarrow x^2 - 4x + 5 = 0$

$y = (x - 2)^2 + 5 \rightarrow (x - 2)^2 = y - 5 \rightarrow y - 5 \geq 0 \rightarrow y \geq 5 \rightarrow R_f = [5, +\infty)$

$x - 2 = \pm \sqrt{y - 5} \rightarrow x = \pm \sqrt{y - 5} + 2$

$x - \frac{5}{2} = \pm \sqrt{y + \frac{1}{4}} \geq 0 \rightarrow R_f = [-\frac{1}{4}, +\infty)$

ب) $y = x^2 - 5x + 1 \rightarrow (x - \frac{5}{2})^2 - (\frac{5}{2})^2 = (x - \frac{5}{2})^2 - \frac{25}{4} \rightarrow y = (x - \frac{5}{2})^2 - \frac{25}{4} \rightarrow (x - \frac{5}{2})^2 = y + \frac{25}{4}$

الف) $y = \frac{x^2 + 3}{x^2 - 2} = yx^2 - 2y = x^2 + 3 \rightarrow yx^2 - x^2 = 2y + 3 \rightarrow x^2(y - 1) = 2y + 3$

$x^2 = \frac{2y + 3}{y - 1} \rightarrow x = \pm \sqrt{\frac{2y + 3}{y - 1}} \rightarrow \frac{-1.5}{+4.5} \rightarrow R_f = (-\infty, -1.5] \cup (1, +\infty)$

ب) $y = \frac{2|x| + 1}{|x| - 4} \rightarrow \ln|y - 2| = 4y + 1 \rightarrow \ln|y - 2| = 4y + 1 \rightarrow |y - 2| = \frac{4y + 1}{y - 2} \geq 0 \rightarrow R = (2, +\infty) \cup (-\infty, -1/4]$

$y = \frac{1}{x^2 - 4x + 4} = \frac{1}{(x - 2)^2} \rightarrow \frac{1}{(x - 2)^2} = y \rightarrow (x - 2)^2 = \frac{1}{y} \rightarrow x - 2 = \pm \sqrt{\frac{1}{y}} \rightarrow x = 2 \pm \sqrt{\frac{1}{y}} \rightarrow R_f = (0, +\infty)$

$yx^2 - 4xy = 1 \rightarrow yx^2 - 4y = 1 \rightarrow y(x^2 - 4) = 1 \rightarrow x^2 - 4 = \frac{1}{y} \rightarrow x^2 = \frac{1}{y} + 4 \rightarrow x = \pm \sqrt{\frac{1}{y} + 4} \rightarrow R_f = (-\infty, -1/4] \cup (0, +\infty)$

$\Delta = 16y^2 + 4y \geq 0 \rightarrow 4y(4y + 1) \geq 0 \rightarrow y = -1/4, y = 0$

الف) $y = x^2 - 4x + 2 \rightarrow \min, \rightarrow [-4, +\infty)$

طول = $\frac{4}{2} = 2$

عرض = -4

ب) $y = -x^2 + 4x + 2 \rightarrow \max, \rightarrow (-\infty, 4]$

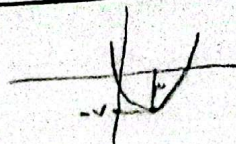
طول = $\frac{-4}{-2} = 2$

عرض = 4

الف) $y = \sqrt{x^2 - 4x + 2} \rightarrow \left\{ \begin{array}{l} \frac{-b}{2a} = \frac{4}{2} = 2 \\ -\sqrt{\Delta} \end{array} \right\} \text{ min دار}$

$\sqrt{[-\infty, +\infty)} \rightarrow R_f = [0, +\infty)$

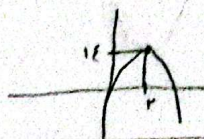
$[-\infty, +\infty)$



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ب) $y = \sqrt{-x^2 + 4x + 1} \rightarrow \text{max دار}$

$\frac{-b}{2a} = 2 \rightarrow \sqrt{(-\infty, 4]} \rightarrow R_f = [0, \sqrt{17}]$



الف) $x^2 + 3x + 2 \rightarrow \text{بزرگترین توان فرد} \rightarrow R_f = \mathbb{R}$

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ب) $y = \sqrt{x^2 + 4x + 4} \rightarrow \sqrt{R} \rightarrow R_f = \mathbb{R}^+ \cup \{0\} \rightarrow [0, +\infty)$

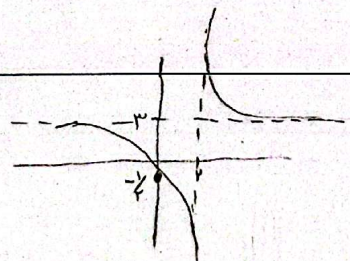
الف) $y = \frac{3x+1}{x-2} \rightarrow R_f = \mathbb{R} - \{3\}$

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ب) $\sqrt{\frac{4x+1}{x+2}} = \frac{a}{c} = 2 \rightarrow R_f = \mathbb{R} - \{2\} = \mathbb{R}^+ \cup \{0\} - \{2\} \Rightarrow [0, +\infty) - \{2\}$

الف) $y = \frac{3x+1}{x-2}$

$x=0 \rightarrow y = -\frac{1}{2}$

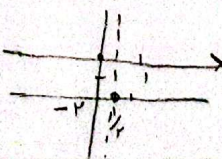


مخانب قائم $\leftarrow$ همیشه مخرج $\leftarrow x=2$

مخانب افق $\leftarrow y = \frac{a}{c} \leftarrow y=3$

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ب) $\frac{4x-2}{1-2x} \xrightarrow{\text{غیر معمولی}} ad=bc \rightarrow \mathbb{R} - \{\frac{1}{2}\}$



الف) $y = \frac{\cos^2 x}{1} + \frac{1}{\cos^2 x} \rightarrow R_f = [2, +\infty)$

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ب) $y = \sqrt{\frac{x^2+1}{x}} \rightarrow \sqrt{x + \frac{1}{x}} \rightarrow (-\infty, -2] \cup [2, +\infty)$