(1) (الف)

$y = x^2 - 5 \longrightarrow x^2 = y + 5 \longrightarrow x = \pm\sqrt{y+5}$ $\quad R_f = [-5, +\infty)$

(ب)

$y = x^3 + 1 \longrightarrow x^3 = y - 1 \longrightarrow x = \sqrt[3]{y-1}$ $\quad R_f = \mathbb{R}$

(2) (الف)

$y = x^2 - 4x + 6 \longrightarrow y = x^2 - 4x + 2^2 + 2$

$y = (x-2)^2 + 2 \longrightarrow (x-2)^2 = y - 2$

$x = \pm\sqrt{y-2} + 2$ $\quad R_f = [2, +\infty)$

(ب)

$y = x^2 - 5x + 1 \longrightarrow y = (x - \frac{5}{2})^2 - \frac{21}{4}$

$(x - \frac{5}{2})^2 = y + \frac{21}{4} \longrightarrow x = \pm\sqrt{y + \frac{21}{4}} + \frac{5}{2}$ $\quad R_f = [-\frac{21}{4}, +\infty)$

(3) (الف)

$yx^2 - 2y = x^2 + 3 \longrightarrow yx^2 - x^2 = 3 + 2y$

$x^2(y-1) = 3 + 2y$

$x^2 = \frac{3+2y}{y-1} \longrightarrow x = \pm\sqrt{\frac{3+2y}{y-1}}$

$R_f = (1, +\infty) \cup (-\infty, -\frac{3}{2}]$

$$\begin{array}{c|c|c|c|c} & -\frac{3}{2} & 0 & 1 & \\ \hline + & 0 & - & \text{تعریف نشده} & + \end{array}$$

(ب)

$y|x| - 4y = 2|x| + 1 \longrightarrow y|x| - 2|x| = 4y + 1$

$|x|(y-2) = 4y + 1$

$|x| = \frac{4y+1}{y-2}$ $\quad R_f = (-\infty, -\frac{1}{4}] \cup (2, +\infty)$

$$\begin{array}{c|c|c|c|c} & -\frac{1}{4} & 0 & 2 & \\ \hline + & 0 & - & \text{تعریف نشده} & + \end{array}$$

(۴)

$y = \dfrac{1}{x^2 - 4x} \longrightarrow y = \dfrac{1}{x^2 - 4x + 4}$

$(x-2)^2 y = 1$

$x - 2 = \pm\sqrt{1-y} \longrightarrow x = \pm\sqrt{1-y} + 2$

$+ \quad 1 \quad -$

$R_f = (-\infty, 1]$

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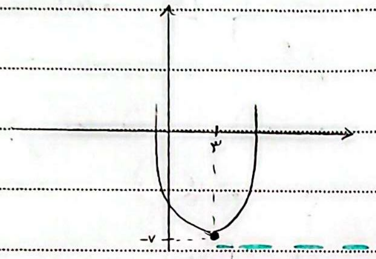
(۵) الف)

$y = x^2 - 6x + 2 \longrightarrow \underline{\text{ext min}} \begin{cases} x = \dfrac{-b}{2a} \to -\left(\dfrac{-6}{2}\right) = 3 \\ 9 - 18 + 2 = -7 \end{cases}$

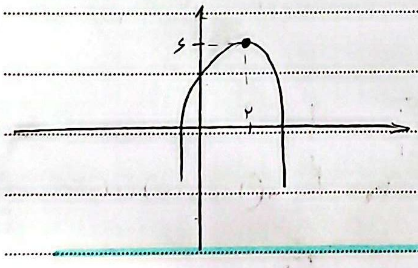
$R_f = [-7, +\infty)$



ب)

$y = -x^2 + 4x + 2 \longrightarrow \underline{\text{ext max}} \begin{cases} x = \dfrac{-b}{2a} \to -\left(\dfrac{4}{-2}\right) = +2 \\ -4 + 8 + 2 = 6 \end{cases}$

$R_f = (-\infty, 6]$



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(۶) الف)

$y = \sqrt{x^2 - 6x + 2}$ $\quad \underline{\text{ext Min}} \begin{cases} x = \dfrac{-b}{2a} = -\left(\dfrac{-6}{2}\right) = +3 \\ y = 9 - 18 + 2 = -7 \end{cases}$

$R_f = \sqrt{[-7, +\infty)} = [0, +\infty)$

ب)

$y = \sqrt{-x^2 + 4x + 10}$ $\quad \text{ext Max} \begin{cases} x = \dfrac{-b}{2a} = -\left(\dfrac{4}{-2}\right) = +2 \\ -4 + 8 + 10 = +14 \end{cases}$

$R_f = \sqrt{(-\infty, 14]} = [0, \sqrt{14}]$

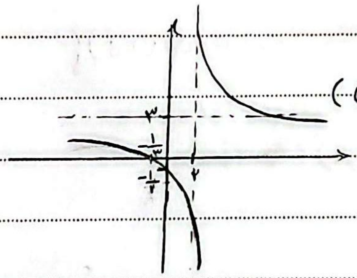
۷) الف) بزرگترین توان فرد است، پس $R_f = R$

ب) $R_f = \sqrt{R} = [0, +\infty)$

۸) الف) $R_f = R - \{3\}$

ب) $R_f = R - \{2\}$ $\sqrt{\frac{4}{1}} = 2$

۹) الف)



$y = \frac{3x+1}{x-2}$

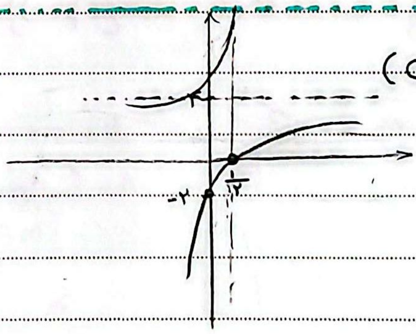
$x = 2$ = مجانب قائم

$y = \frac{a}{c} = 3$ = مجانب افقی

برخورد با x $\Rightarrow y=0 \rightarrow -\frac{1}{3}$

برخورد با y $\Rightarrow x=0 \rightarrow -\frac{1}{2}$

ب)



$y = \frac{4x-2}{1-2x}$

$x = \frac{1}{2}$ = مجانب قائم

$y = \frac{a}{c} = 4$ = مجانب افقی

برخورد با x $\Rightarrow y=0 \rightarrow \frac{1}{2}$

برخورد با y $\Rightarrow x=0 \rightarrow -2$

۱۰) الف) چون توان ۲ دارد cos همیشه + می شود پس $R_f = [2, +\infty)$

ب) $\sqrt[3]{\frac{x^2+1}{x}} = \sqrt[3]{x+\frac{1}{x}} \Rightarrow \left.\begin{matrix} x>0 \\ x<0 \end{matrix}\right\} \rightarrow R_f = (-\infty, -2] \cup [2, +\infty)$