آیلا جعفری - دهم دفتر A - تکلیف شماره ۹

۱-

الف) $y = x^2 - 5 \rightarrow x^2 = 5 + y \rightarrow x = \pm\sqrt{5+y} \rightarrow R_f = [-5, +\infty)$

ب) $y = x^3 + 1 \rightarrow x^3 = -1 + y \rightarrow x = \sqrt[3]{-1+y} \rightarrow R_f = \mathbb{R}$

۲-

الف) $y = x^2 - 4x + 6 = x^2 - 4x + 4 + 2 = (x-2)^2 + 2 \rightarrow (x-2)^2 = y - 2 \rightarrow x - 2 = \pm\sqrt{y-2} \rightarrow$
$x = \pm\sqrt{y-2} + 2 \rightarrow R_f = [2, +\infty)$

$R_f = [\frac{-21}{4}, +\infty)$

ب) $y = x^2 - 5x + 1 \rightarrow x^2 - 5x + (1-y) = 0$
$ax^2 + bx + c \rightarrow \Delta \geq 0 \rightarrow \Delta = (-5)^2 - 4(1)(1-y) \geq 0 \rightarrow 21 + 4y \geq 0 \rightarrow 4y \geq -21 \rightarrow y \geq \frac{-21}{4}$

۳-

الف) $y = \frac{x^2+3}{x^2-2} \rightarrow yx^2 - 2y = x^2 + 3 \rightarrow yx^2 - x^2 = 2y + 3 \rightarrow x^2(y-1) = 2y + 3 \rightarrow x^2 = \frac{2y+3}{y-1} \rightarrow$
$x = \pm\sqrt{\frac{2y+3}{y-1}} \rightarrow$ $-\frac{3}{2}$, 1 : $+$ $-$ $+$ $\rightarrow R_f = (-\infty, -\frac{3}{2}] \cup (1, +\infty)$

ب) $y = \frac{2|x|+1}{|x|-4} \rightarrow y|x| - 4y = 2|x| + 1 \rightarrow y|x| - 2|x| = 4y + 1 \rightarrow |x|(y-2) = 4y + 1 \rightarrow$
$|x| = \frac{4y+1}{y-2} \rightarrow$ $-\frac{1}{4}$, 2 : $+$ $-$ $+$ $\rightarrow R_f = (-\infty, -\frac{1}{4}] \cup (2, +\infty)$

۴-

$y = \frac{1}{x^2-4x} \rightarrow yx^2 + 4xy = 1 \rightarrow yx^2 + 4yx - 1 = 0 \rightarrow \Delta = b^2 - 4ac = (4y)^2 - 4(y)(-1) \geq 0$
$\rightarrow y = -\frac{1}{4}, y = 0 \rightarrow R_f = (-\infty, -\frac{1}{4}] \cup (0, +\infty)$

چون $\frac{1}{x^2+4x}$ صفر نمی‌شود.

۵-

الف) $y = x^2 - 6x + 2 \xrightarrow[\text{min دارد}]{a>0} ext \left| \begin{array}{l} \frac{-b}{2a} = \frac{6}{2} = 3 \\ -7 \end{array} \right. \rightarrow R_f = [-7, +\infty)$

ب) $y = -x^2 + 4x + 2 \xrightarrow[\text{max دارد}]{a<0} ext \left| \begin{array}{l} \frac{-4}{-2} = 2 \\ 6 \end{array} \right. \rightarrow R_f = (-\infty, 6]$

۶-

الف) $y = \sqrt{x^2 - 6x + 2}$ — $ext \left| \begin{array}{l} 3 \\ -7 \end{array} \right. \rightarrow R_f = \sqrt{[-7, +\infty)} = [0, +\infty)$

ب) $y = \sqrt{-x^2 + 4x + 10} \xrightarrow[\text{max دارد}]{a<0} ext \left| \begin{array}{l} \frac{-b}{2a} = \frac{-4}{-2} = 2 \\ 14 \end{array} \right. \rightarrow R_f = \sqrt{(-\infty, 14]} = [0, \sqrt{14}]$

۷- الف) $y = x^7 + 3x^4 + 2x + 1 \rightarrow$ چون بزرگترین توانش فرده.

پس: $R_f = \mathbb{R}$

ب) $y = \sqrt{x^5 + 4x^2 + 6x + 1} \rightarrow R_f = \sqrt{\mathbb{R}} = [0, +\infty)$

باید نامنفی باشد.

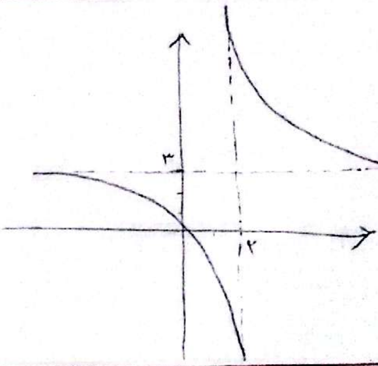
۸- الف) $y = \dfrac{3x+1}{x-2} \rightarrow \dfrac{a}{c} = \dfrac{3}{1} = 3 \rightarrow R_f = \mathbb{R} - \{3\}$

ب) $y = \sqrt{\dfrac{4x+1}{x+3}} \longrightarrow \dfrac{a}{c} = \dfrac{4}{1} = 4 \rightarrow R_f = [0, +\infty) - \{2\}$

$\sqrt{4} = 2$

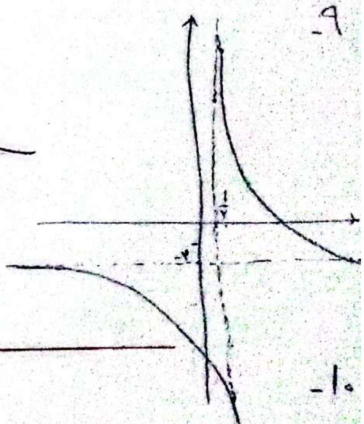
۹- الف) $y = \dfrac{3x+1}{x-2} \rightarrow$ 2 = ریشه مخرج = مجانب قائم

افقی " = $\dfrac{a}{c} = \dfrac{3}{1} = 3$



ب) $y = \dfrac{4x-2}{1-2x} \rightarrow$ $\dfrac{1}{2}$ = ریشه مخرج = مجانب قائم

افقی " = $\dfrac{a}{c} = -2 = \dfrac{4}{-2}$



۱۰- الف) $y = \cos^2 x + \dfrac{1}{\cos^2 x} \rightarrow R_f = [2, +\infty)$

ب) $y = \sqrt[3]{\dfrac{x^2+1}{x}} = \sqrt[3]{x + \dfrac{1}{x}} \rightarrow R_f = \mathbb{R}$