

١) الف) $y = a^2 - \Delta \Rightarrow a^2 = y + \Delta \quad a = \pm \sqrt{y + \Delta} \quad R_f = [-\infty, +\infty)$

$y + \Delta \geq 0$
 $y \geq -\Delta$

ب) $a^2 = 1 \Rightarrow a^2 = y - 1 \quad a = \pm \sqrt{y - 1} \Rightarrow R_f = \mathbb{R}$

٢) الف) $y = a^2 - \varepsilon a + r \quad R_f = [r, +\infty)$

$y = a^2 - \varepsilon a + r - r + r$

$y = (a - r)^2 + r$

$(a - r)^2 = y - r$

$a - r = \pm \sqrt{y - r} \Rightarrow a = \pm \sqrt{y - r} + r$

$y \geq r$
 $y - r \geq 0$

ب) $y = a^2 - \varepsilon a + 1$

$a^2 - \varepsilon a + 1 - y = 0$

$\Delta = \varepsilon^2 - 4(1 - y)$

$\Delta = \varepsilon^2 - 4 + 4y$
 $\Delta = \varepsilon^2 - 4 + 4y$

$a = \frac{\varepsilon \pm \sqrt{\varepsilon^2 - 4 + 4y}}{2}$

$R_f = \left[\frac{\varepsilon^2}{4}, +\infty \right)$

$\varepsilon^2 - 4 + 4y \geq 0 \quad \varepsilon^2 \geq 4 - 4y \quad \varepsilon \geq \frac{\varepsilon^2}{2}$

٣) الف) $y = \frac{a^2 + r}{a^2 - r} \Rightarrow y(a^2 - r) = a^2 + r$

$ya^2 - yr = a^2 + r \Rightarrow a^2(1 - y) = -yr - r$

$a^2 = \frac{-yr - r}{1 - y} \Rightarrow a^2 = \frac{ry + r}{y - 1}$

$a = \pm \sqrt{\frac{ry + r}{y - 1}}$

$R_f = (-\infty, \frac{r}{1}] \cup (1, +\infty)$

$\frac{-r}{1} \leq 1$

$ry + r \geq 0$
 $ry \geq -r$
 $y \geq \frac{-r}{r}$

$y - 1 \leq 0$
 $y \leq 1$

ب) $y = \frac{r|a| + 1}{|a| - \varepsilon} \Rightarrow y(|a| - \varepsilon) = r|a| + 1$

$\frac{-1}{\varepsilon} \leq 1$

$r|a| - y|a| = -\varepsilon y - 1 \Rightarrow |a|(r - y) = -\varepsilon y - 1$

$|a| = \frac{\varepsilon y + 1}{y - r}$

$R_f = (-\infty, \frac{1}{\varepsilon}] \cup (r, +\infty)$

$y - r \leq 0 \quad \varepsilon y + 1 \geq 0$
 $y \leq r \quad \varepsilon y \geq -1$
 $y \geq \frac{-1}{\varepsilon}$

٤) $y = \frac{1}{a^2 - \varepsilon a} \Rightarrow y(a^2 - \varepsilon a) = 1 \Rightarrow ya^2 - \varepsilon ya - 1 = 0$

$a = \frac{\varepsilon y \pm \sqrt{\varepsilon^2 y^2 + 4y}}{2y}$

$\varepsilon^2 y^2 + 4y \geq 0 \Rightarrow \varepsilon^2 y^2 \geq -4y$
 $y \geq \frac{-1}{\varepsilon}$

$R_f = (-\infty, \frac{1}{\varepsilon}] \cup [\omega, +\infty)$

سوال ٥) الف) $y = ax^2 - 6ax + 2 \Rightarrow a > 0 \quad \begin{cases} x_{min} = \frac{-b}{2a} = \frac{+6}{2} = 3 \\ y_{min} = -V \end{cases} \quad R_f = [-V, +\infty)$

ب) $y = -ax^2 + 6ax + 2 \quad a < 0 \quad \begin{cases} x_{max} = \frac{-b}{2a} = \frac{-6}{-2} = 3 \\ y_{max} = 6 \end{cases} \quad R_f = (-\infty, 6]$

سوال ٦) الف) $y = \sqrt{ax^2 - 6ax + 2} \quad a > 0 \quad \begin{cases} x_{min} = \frac{+6}{2} = 3 \\ y_{min} = -V \end{cases} \quad R_f = \sqrt{[-V, +\infty)} = R_f = [0, +\infty)$

ب) $y = \sqrt{-ax^2 + 6ax + 2} \quad a < 0 \quad \begin{cases} x_{max} = 3 \\ y_{max} = 12 \end{cases} \quad R_f = \sqrt{(-\infty, 12]} \Rightarrow R_f = [0, \sqrt{12}]$

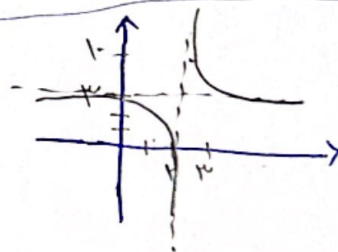
سوال ٧) الف) $y = ax^4 + 4ax^3 + 6ax^2 + 1 \rightarrow R_f = \mathbb{R}$

ب) $y = \sqrt{ax^4 + 4ax^3 + 6ax^2 + 1} \rightarrow R_f = \sqrt{\mathbb{R}} = [0, +\infty)$

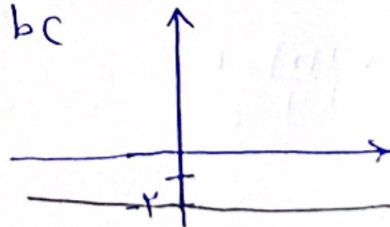
سوال ٨) الف) $y = \frac{2ax+1}{a-2} \quad R_f = \mathbb{R} - \{2\}$
 $\frac{a}{c} = 2$

ب) $y = \frac{2ax+1}{\sqrt{a+2}} \quad R_f = [0, +\infty) - \{2\}$

سوال ٩) الف) $y = \frac{2ax+1}{a-2} \quad \begin{matrix} a-2=0 \\ a=2 \end{matrix} \rightarrow \begin{matrix} \text{جيب} \\ \text{ف} \\ \text{ف} \end{matrix} \rightarrow \begin{matrix} \text{نقطة كسري} \\ \text{نقطة كسري} \end{matrix} = \left[\frac{2}{1} \right]$
 $\hat{a} = 2$ جانب ايسر



ب) $y = \frac{2ax-2}{1-2a} \rightarrow \frac{2(2a-1)}{-(2a-1)} = -2$
 $-2(-2) = 4 = f(1) \quad ad \neq bc$
 $y = \frac{2(2a-1)}{-(2a-1)} = -2$



سوال ١٠) الف) $y = \cos^2 x + \frac{1}{\cos^2 x} \quad \cos^2 x > 0 \rightarrow R_f = [2, +\infty)$

ب) $y = \sqrt{\frac{a^2+1}{a}} \Rightarrow \sqrt{a + \frac{1}{a}} \Rightarrow R_f = (-\infty, -1] \cup [1, +\infty)$
 $a^2 \leq a \Rightarrow a \leq 1$