

الف) $y = x^2 - \omega \Rightarrow x^2 = y + \omega \rightarrow x = \pm \sqrt{y + \omega} \rightarrow y + \omega \geq 0$

$y \geq -\omega$

$R_f = [-\omega, +\infty)$

ب) $y = x^3 + 1 \Rightarrow x^3 = y - 1 \rightarrow x = \sqrt[3]{y - 1}$

$R_f = \mathbb{R}$

الف) $y = x^2 - \varepsilon x + \gamma \Rightarrow y = x^2 - \varepsilon x + \varepsilon + \gamma \rightarrow y = (x - \frac{\varepsilon}{2})^2 + \gamma$

$\rightarrow (x - \frac{\varepsilon}{2})^2 = y - \gamma \rightarrow x - \frac{\varepsilon}{2} = \pm \sqrt{y - \gamma} \rightarrow x = \pm \sqrt{y - \gamma} + \frac{\varepsilon}{2} \rightarrow y - \gamma \geq 0 \rightarrow y \geq \gamma$

$R_f = [\gamma, +\infty]$

ب) $y = x^2 - \alpha x + 1 \Rightarrow y = x^2 - \alpha x + \frac{\alpha^2}{4} - \frac{\alpha^2}{4} + 1 \rightarrow y = (x - \frac{\alpha}{2})^2 - \frac{\alpha^2}{4} + 1 \rightarrow (x - \frac{\alpha}{2})^2 = y + \frac{\alpha^2}{4} - 1$
 $\rightarrow x - \frac{\alpha}{2} = \pm \sqrt{y + \frac{\alpha^2}{4} - 1} \rightarrow x = \pm \sqrt{y + \frac{\alpha^2}{4} - 1} + \frac{\alpha}{2} \rightarrow y + \frac{\alpha^2}{4} - 1 \geq 0 \rightarrow y \geq -\frac{\alpha^2}{4} + 1$
 $R_f = [-\frac{\alpha^2}{4} + 1, +\infty)$

الف) $y = \frac{x^2 + 3}{x^2 - 2} \Rightarrow y(x^2 - 2) = x^2 + 3 \rightarrow yx^2 - x^2 = 3 + 2y = x^2(y - 1) = 3 + 2y$
 $x^2 = \frac{3 + 2y}{y - 1} = x = \pm \sqrt{\frac{3 + 2y}{y - 1}} \geq 0 \rightarrow \frac{3 + 2y}{y - 1} \geq 0$

ب) $y = \frac{2|x| + 1}{|x| - \varepsilon} \Rightarrow y(|x| - \varepsilon) = 2|x| + 1 \rightarrow y|x| - \varepsilon y = 2|x| + 1 \rightarrow |x|(y - 2) = \varepsilon y + 1$
 $\rightarrow |x| = \frac{\varepsilon y + 1}{y - 2} \rightarrow x = \pm \frac{\varepsilon y + 1}{y - 2} \rightarrow \frac{\varepsilon y + 1}{y - 2} \geq 0$
 $R_f = (-\infty, -\frac{1}{\varepsilon}] \cup (2, +\infty)$

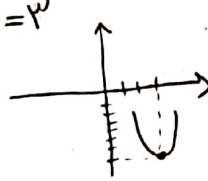
$y = \frac{1}{x^2 - \varepsilon x} \Rightarrow y(x^2 - \varepsilon x) = 1 \rightarrow yx^2 - \varepsilon yx - 1 = 0$
 $\Delta = b^2 - 4ac = \varepsilon^2 y^2 + 4y$
 $x = \frac{\varepsilon y \pm \sqrt{\varepsilon^2 y^2 + 4y}}{2y}$
 $\Delta \geq 0 \rightarrow \varepsilon^2 y^2 + 4y \geq 0 \rightarrow y(\varepsilon^2 y + 4) \geq 0 \rightarrow y \geq -\frac{4}{\varepsilon^2}$
 $R_f = [-\frac{4}{\varepsilon^2}, +\infty)$

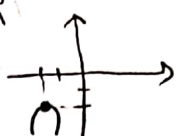
الف) $y = x^2 - 4x + 2 \Rightarrow \text{ext} \begin{cases} x_{\min} = \frac{-b}{2a} = \frac{4}{2} = 2 \\ y_{\min} = -\sqrt{\quad} \end{cases}$

$R_f = [y_{\min}, +\infty)$

ب) $y = -x^2 + \varepsilon x + 2 \Rightarrow \text{ext} \begin{cases} x_{\max} = \frac{b}{2a} = \frac{\varepsilon}{-2} = -\frac{\varepsilon}{2} \\ y_{\max} = -\sqrt{\quad} \end{cases}$

$R_f = (-\infty, y_{\max}]$
 $R_f = (-\infty, -2]$

الف) $y = \sqrt{x^2 - 4x + 4} \Rightarrow \text{ext} \begin{cases} x_{\min} = \frac{-b}{2a} = \frac{4}{2} = 2 \\ y_{\min} = -1 \end{cases}$  $R_f = \sqrt{[-1, +\infty)}$
 $R_f = [0, +\infty)$

ب) $y = \sqrt{-x^2 + 8x + 10} \Rightarrow \text{ext} \begin{cases} x_{\max} = \frac{-b}{2a} = \frac{-8}{-2} = 4 \\ y_{\max} = 14 \end{cases}$  $R_f = \sqrt{(-\infty, 14]}$
 $R_f = [0, \sqrt{14}]$

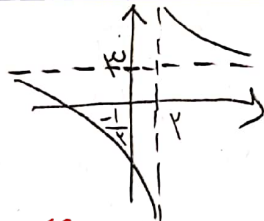
الف) $y = x^5 + 3x^4 + 4x + 1 \Rightarrow R_f = (-\infty, +\infty) \subseteq R_f = \mathbb{R}$

ب) $y = \sqrt{x^5 + 8x^4 + 4x + 1} \Rightarrow R_f = [0, +\infty)$

الف) $y = \frac{3x+1}{x-2} \Rightarrow y = \frac{ax+b}{cx+d} \rightarrow ad \neq cb \rightarrow R_f = \mathbb{R} - \left\{\frac{a}{c}\right\}$
 $R_f = \mathbb{R} - \{3\}$

ب) $y = \sqrt{\frac{3x+1}{x+3}} \Rightarrow y = \frac{ax+b}{cx+d} \rightarrow ad \neq cb \rightarrow R_f = [0, +\infty) - \left\{\sqrt{\frac{a}{c}}\right\}$
 $\rightarrow R_f = [0, +\infty) - \{2\}$ چون زیر رادیکال است

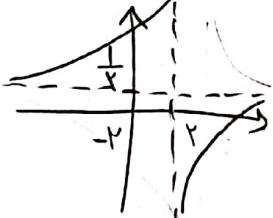
الف) $y = \frac{3x+1}{x-2}$



مجاذب قائم $y=3$ و $x=2$ $y=3$ $x=2$ $\frac{a}{c} = \frac{3}{1} = 3$

ب) $y = \frac{3x-2}{1-2x}$

همواره مثبت نیست $\frac{2(2x-1)}{-(2x-1)} = -2$



مجاذب قائم $y=1.5$ و $x=0.5$ $\frac{a}{c} = \frac{3}{-2} = -1.5$

الف) $y = \cos^2 x + \frac{1}{\cos^2 x} \Rightarrow y = a + \frac{1}{a} \xrightarrow{a>0} [2, +\infty)$

$R_f = [2, +\infty)$

ب) $y = \sqrt[3]{\frac{x^2+1}{x}} \Rightarrow y = \sqrt[3]{x + \frac{1}{x}} \xrightarrow{a<0} \sqrt[3]{(-\infty, -2] \cup [2, +\infty)}$

$R_f = (-\infty, -\sqrt[3]{2}] \cup [\sqrt[3]{2}, +\infty)$