

الف) $y = x^2 - a \rightarrow x^2 = y + a \quad x = \pm \sqrt{y+a} \quad R_f = [-a, +\infty)$

(1)

ب) $y = x^2 + 1 \rightarrow x^2 = y - 1 \quad x = \pm \sqrt{y-1} \quad R_f = \mathbb{R}$

الف) $y = x^2 - 2x + 4 \rightarrow y = x^2 - 2x + 1 - 1 + 4 \rightarrow y = (x-1)^2 - 1 + 4 \quad y = (x-1)^2 + 3 \rightarrow (x-1)^2 = y - 3$

(2)

$x-1 = \pm \sqrt{y-3} \rightarrow x = \pm \sqrt{y-3} + 1 \quad R_f = [3, +\infty)$
 $y-3 \geq 0 \rightarrow y \geq 3$

ب) $y = x^2 - 2x + 1 \rightarrow y = x^2 - 2x + \frac{16}{4} - \frac{16}{4} + 1 \rightarrow y = (x-\frac{1}{2})^2 - \frac{16}{4} + \frac{5}{4} \rightarrow y = (x-\frac{1}{2})^2 - \frac{11}{4} \rightarrow (x-\frac{1}{2})^2 = y + \frac{11}{4}$

$x - \frac{1}{2} = \pm \sqrt{y + \frac{11}{4}} \rightarrow x = \pm \sqrt{y + \frac{11}{4}} + \frac{1}{2} \quad R_f = [-\frac{11}{4}, +\infty)$
 $y + \frac{11}{4} \geq 0 \rightarrow y \geq -\frac{11}{4}$

الف) $y = \frac{x^2 + 1}{x^2 - 2} \rightarrow y(x^2 - 2) = x^2 + 1 \rightarrow yx^2 - 2y = x^2 + 1 \rightarrow yx^2 - x^2 = 1 + 2y \quad x^2(y-1) = 1+2y \rightarrow x^2 = \frac{1+2y}{y-1}$

(3)

$\frac{-\frac{1}{2}}{+} \quad \frac{1}{-}$

$x = \pm \sqrt{\frac{1+2y}{y-1}} \quad R_f = (-\infty, -\frac{1}{2}] \cup (1, +\infty)$

ب) $y = \frac{2|x|+1}{|x|-2}$

$y|x| - 2y = 2|x| + 1 \rightarrow y|x| - 2|x| = 1 + 2y \rightarrow |x|(y-2) = 1+2y \quad |x| = \frac{1+2y}{y-2}$

$\frac{-\frac{1}{2}}{+} \quad \frac{1}{-}$

$R_f = (-\infty, -\frac{1}{2}] \cup (2, +\infty)$

$y = \frac{1}{x^2 - 1}$

$y(x^2 - 1) = 1 \rightarrow yx^2 - 1 = 1 \rightarrow yx^2 - 2 = 0$

$x = \frac{-b \pm \sqrt{\Delta}}{2a} \rightarrow \Delta \geq 0 \rightarrow 4y^2 + 4 \geq 0$

(4)

$\frac{-\frac{1}{2}}{+} \quad \frac{1}{-}$

$4y(4y+1) \geq 0 \quad R_f = (-\infty, -\frac{1}{4}] \cup [0, +\infty)$

الف) $y = x^2 - 4x + 4 \xrightarrow{a > 0} \begin{cases} x_{\min} = \frac{-b}{2a} = 2 \\ y_{\min} = -4 \end{cases}$

$R_f = [-4, +\infty)$

(5)

ب) $y = -x^2 + 4x + 4 \xrightarrow{a < 0} \begin{cases} x_{\max} = \frac{-b}{2a} = 2 \\ y_{\max} = 4 \end{cases}$

$R_f = (-\infty, 4]$

$$\text{الف) } y = \sqrt{x^2 - 4x + 4} \xrightarrow{a \geq 0} \begin{cases} x_{\min} = \frac{-b}{2a} = 2 \\ y_{\min} = -2 \end{cases}$$

$$R_f = \sqrt{[-2, +\infty)} = [0, +\infty)$$

⑥

$$\text{ب) } y = \sqrt{-x^2 + 6x + 10} \xrightarrow{a < 0} \begin{cases} x_{\max} = \frac{-b}{2a} = 3 \\ y_{\max} = 4 \end{cases}$$

$$R_f = \sqrt{(-\infty, 4]} = [0, \sqrt{14}]$$

$$\text{الف) } y = x^2 + 12x + 1 \quad R_f = \mathbb{R}$$

⑦

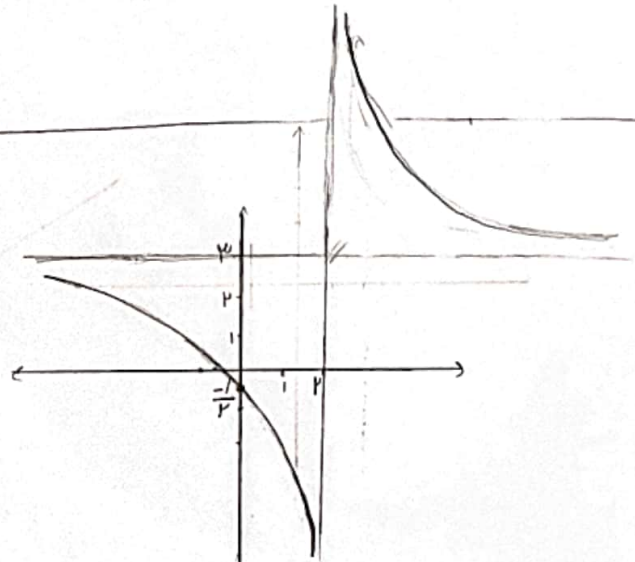
$$\text{ب) } y = \sqrt{x^2 + 6x + 10} \quad R_f = [0, +\infty)$$

$$\text{الف) } y = \frac{3x+1}{x-2} \quad R_f = \mathbb{R} - \{2\}$$

⑧

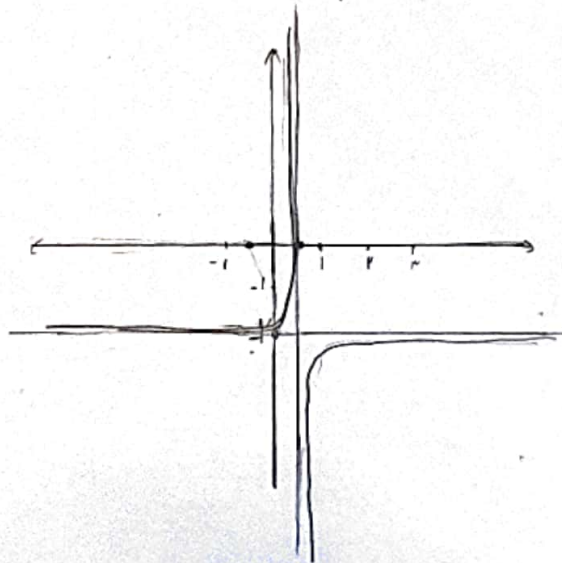
$$\text{ب) } y = \sqrt{\frac{3x+1}{x+2}} \quad R_f = [0, +\infty) - \{2\}$$

$$\text{الف) } y = \frac{3x+1}{x-2} \quad \begin{aligned} & \text{مخرج صفر} \rightarrow x = 2 \\ & \text{جانب راست} \rightarrow y = \frac{a}{c} = \frac{3}{-2} \\ & \text{تقاطع محور} \rightarrow 3x+1=0 \rightarrow x = -\frac{1}{3} \end{aligned}$$



⑨

$$\text{ب) } y = \frac{3x-2}{1-2x} \quad \begin{aligned} & \text{مخرج صفر} \rightarrow x = \frac{1}{2} \\ & \text{جانب راست} \rightarrow y = \frac{a}{c} = -2 \\ & \text{تقاطع محور} \rightarrow 3x-2=0 \rightarrow x = \frac{2}{3} \end{aligned}$$



(10)

$$\text{الف) } y = \cos^r x + \frac{1}{\cos^r x} \quad R_f = [r, +\infty)$$

$$\text{ب) } y = \sqrt[r]{\frac{x^r + 1}{x}} \quad R_f = (-\infty, \sqrt[r]{-r}] \cup [\sqrt[r]{r}, +\infty) \leftarrow \sqrt[r]{x + \frac{1}{x}}$$

$$\sqrt[r]{1}$$