

الف) $f(x) = \sqrt{x|x|}$, $g(x) = x$; $D_f = x|x| \geq 0 \rightarrow x \geq 0 \rightarrow D_f = [0, +\infty)$, $D_g = \mathbb{R}$ * تآویز دارند

ج) $f(x) = \frac{(x+r)(x+1)+x+r}{x^2+px+q}$, $g(x)=1$; $f(x) = \frac{x^2+px+r+x+r}{x^2+px+q} = 1$, $g(x)=1 \Rightarrow D_f = D_g = \mathbb{R}$ * برای درج

$$2.) f(x) = \frac{r \sin^2 x - 4 \sin x}{r \sin x - 4}, \quad g(x) = r \sin x; \quad f(x) = \frac{r \sin x (r \sin x - 4)}{r \sin x - 4} = r \sin x, \quad g(x) = r \sin x \quad Df = Dg = 1R$$

2) $f(x) = \frac{x}{|x|} \rightarrow D_f = \mathbb{R} \setminus \{0\}$, $g(x) = \frac{|x|}{x} \rightarrow D_g = \mathbb{R} \setminus \{0\}$; $D_f = D_g$ $a, b \in \mathbb{C}$

مثال ۱: $f(x) = \left[\frac{x}{x+1} \right]$ و $g(x) = \left[x - [x] \right]$; $D_f = D_g = \mathbb{R}$

b) $f(x) = \frac{1}{\sqrt{x}}$, $g(x) = \frac{1}{\sqrt{x}}$; $\frac{1}{\sqrt{x}} \rightarrow [\sqrt{x}] \neq \rightarrow r_2 \neq [0, 1) \rightarrow D_f = (-\infty, 0) \cup [\frac{1}{v}, +\infty)$

$$g(x) = \frac{1}{r[n]} \rightarrow r[n] \neq \cdot \rightarrow [n] \neq \cdot \rightarrow x \notin [0,1] \rightarrow Dg = (-\infty, 0) \cup (1, +\infty)$$

Ex) $f(x) = \frac{1}{\sqrt{x-1}}$, $g(x) = \frac{1}{\sqrt{|x-1|}}$; $f(x) = \frac{1}{\sqrt{x-1}}$ $\frac{+}{-}$ $D_f = \emptyset$, $g(x) = \frac{1}{\sqrt{|x-1|}}$ $\frac{+}{-}$ $D_g = (-\infty, 1) \cup (1, \infty)$

$$2) f(x) = \begin{cases} \frac{x^r - x}{x-1}; & x \neq 1 \\ r; & x = 1 \end{cases}, \quad g(x) = x^r + x; \quad f(n), \quad \frac{x^r - x}{x-1} = \frac{x(x^{r-1} - 1)}{x-1} = x(n+1) = x^r + x \quad Df = IR - \{1\}$$

$$f(x) + g(x) = x^r + x + 1, \quad f(x) - g(x) = x^r - x + 1 \quad ; \quad f'(x) - g'(x) = ? \quad , \quad f(x) = ? \quad , \quad g(x) = ?$$

$$\begin{cases} f(n) + g(n) = n^r + n + 1 \\ f(n) - g(n) = n^r - n + 1 \end{cases} \rightarrow r f(n) = r n^r + r \Rightarrow r f(n) = r(n^r + 1) \rightarrow f(n) = \frac{r}{2} n^r + \frac{r}{2}, g(n) = \frac{r}{2} n$$

$$f^r(u) = (x^r + 1)^r = x^r + 1 + r x^r, \quad g^r(u) = x^r$$

$$f'(n) - g'(n) = x^k + rx^r + 1 - x^r = \underline{x^k + x^r + 1}$$

$$f(x) = x - \sqrt{x^2 - 1}, \quad g(x) = x + \sqrt{x^2 - 1} \quad f \times g = ? \quad \star f \times g = D_f \cap D_g$$

$$Df \Rightarrow x^r - r \geq 0 \rightarrow x^r \geq r \rightarrow \begin{cases} x \geq r \\ x \leq -r \end{cases} ; Df = (-\infty, -r] \cup [r, +\infty)$$

$$Dg \Rightarrow x^r - \epsilon, \dots \rightarrow x^r \geq \alpha ; \quad Dg, (-\infty, -\beta] \cup [\tau, +\infty)$$

$$f \times g = D_f \cap D_g = (-\infty, -r] \cup [r, +\infty)$$

$$f(x) = \frac{ax + r}{x^r - mx + n}, \quad g(x) = \frac{x - b}{x^r - rx - \omega}; \quad am - bn = ?$$

$$x^r - mx + n = r(r x^{r-1} - m x - a) \Rightarrow \underline{r x^{r-1} - m x - a} = r x^{r-1} - r x - ar \rightarrow \begin{cases} 1 = r \rightarrow r = 1 \\ -m = -r \rightarrow -m = -1 \Rightarrow m = 1 \\ n = -ar \rightarrow n = -a \end{cases}$$

$$\underline{ax+ry} = r(x-b) + \frac{1}{r}(x-b) \rightsquigarrow \boxed{a = \frac{1}{r}(x-b)}, \frac{1}{r}b = r \Rightarrow \boxed{b = r}$$

$$am - bn = \left(\frac{1}{r} \times \frac{r}{r} \right) - \left(\frac{-r \times -a}{r} \right) = \frac{r}{r} \angle 10 = \frac{-r \times -a}{r} \int \frac{r}{r} \times \frac{1}{r}$$

$$f(x) = \frac{bx+r}{\lambda x+b}, \quad g(x)=c, \quad \text{Dom} f = \mathbb{R} - \{-a\} \quad \frac{ab}{c} = ?$$

$$\lambda x + b \neq 0 \rightarrow x \neq \frac{-b}{\lambda} \quad (\text{ممنوع}) \Rightarrow a = \frac{-b}{\lambda}$$

$$bx+r = (\lambda x+b)c \Rightarrow bx+r = \lambda xc + bc$$

$$b = \lambda c \quad \& \quad c = \frac{b}{\lambda}, \quad r = bc \rightarrow c = \frac{r}{b} \quad \& \quad r = bc = \lambda c^2 \Rightarrow c^2 = \frac{r}{\lambda} = \frac{1}{\lambda} \Rightarrow c = \pm \frac{1}{\sqrt{\lambda}}$$

$$a = \frac{-b}{\lambda} = \frac{-\lambda c}{\lambda} = -c \rightarrow \frac{ab}{c} = \frac{-cb}{c} = -b \rightsquigarrow \begin{cases} (1) c = \frac{1}{\sqrt{\lambda}} \rightarrow -\lambda \cdot \frac{1}{\sqrt{\lambda}} = -\sqrt{\lambda} \\ (2) c = -\frac{1}{\sqrt{\lambda}} \rightarrow -\lambda \cdot (-\frac{1}{\sqrt{\lambda}}) = \sqrt{\lambda} \end{cases} \quad \frac{ab}{c} = \pm \sqrt{\lambda}$$

$$f^{-1} = \{(-1, r), (r, u), (r, -\omega), (-2, 0)\} \rightsquigarrow f = \{(-1, r), (r, u), (r, -r), (0, \frac{1}{r})\}$$

$$g = \{(r, u), (-1, -r), (-1, -r), (\omega, 0)\} \quad \& \quad \frac{rg}{f+g} = ?$$

$$r, r, -1 : g \circ f \quad (\text{استمرارية})$$

$$\text{ممنوع} \rightarrow r: \frac{\lambda}{\lambda} = 1, \quad r: \frac{1}{\lambda} = \frac{1}{\lambda}, \quad -1: \frac{-\lambda}{-\lambda} = 1$$

$$\text{ممنوع} \Rightarrow R = \{r, r, 1\}$$

$$g = \{(r, -1), (-r, 1), (a - rb, \omega)\}, \quad f = \{(r, a), (-r, \underline{a-rb}), (c, \omega), (r, d)\} \quad d+c = ?$$

$$a = d = -1, \quad \frac{ra - rb}{-r} = 1 \rightarrow -rb = r \Rightarrow b = -r, \quad \frac{a - rb}{-r} = c \rightarrow c = \omega$$

$$d+c = -1 + \omega = \frac{r}{\lambda}$$

$$f(x) = \sqrt{-x^2 + x - m}, \quad g(x) = \{(a, b)\} \quad a+b = ?$$

$$-x^2 + x - m \geq 0 \rightarrow \Delta = b^2 - 4ac = 1 - 4m \quad 1 - 4m = 0 \rightarrow 4m = 1 \rightarrow m = \frac{1}{4}$$

$$-x^2 + x - \frac{1}{4} \rightarrow x = \frac{-b}{2a} = \frac{-1}{-2} = \frac{1}{2}, \quad f(\frac{1}{2}) = \sqrt{0} = 0$$

$$g(x) = \{(\frac{1}{2}, 0)\} \rightarrow a+b = \frac{1}{2} + 0 = \frac{1}{2}$$

$$f(x) = \frac{x^2+a}{x^2+4x+b}, \quad g(x) = \frac{r}{x-c}; \quad a+b+c = ?$$

$$(x^2+a)(x-c) = r(x^2+4x+b) \Rightarrow rx^2 - rx + ac = rx^2 + 4rx + rb \Rightarrow \underline{-rx + ac = 4rx + rb}$$

$$x(a-r) - ac = 4rx + rb \rightarrow a-r = 4r, \quad -ac = rb \rightarrow b = \frac{-ac}{r}$$

$$\star (x-c)^2 : x^2 + 4x + b \rightarrow x^2 - \underline{rc} + c^2 = x^2 + 4x + b \Rightarrow -rc = 4 \Rightarrow \underline{c = -\frac{4}{r}} \quad (\text{P}) \quad b = 9, \quad a - \underline{rc} = 4r \Rightarrow \underline{a = 4} \quad (\text{Q})$$

$$a+b+c = -4 + 9 + 4 = 9 \rightarrow \text{جواب آخر}$$