۱

الف) $f(x)=\sqrt{x|x|}\ ,\ g(x)=x \rightsquigarrow x|x|\geq 0 \rightsquigarrow x\geq 0 \Rightarrow D_f=[0,+\infty)\ ,\ D_g=\mathbb{R} \Rightarrow f(x)\neq g(x)$

ب) $f(x)=\dfrac{(x+3)(x+1)+x+4}{\underbrace{x^2+5x+7}_{\Delta<0}}\ ,\ g(x)=1 \rightsquigarrow f(x)=\dfrac{x^2+5x+7}{x^2+5x+7}=1\ ,\ D_f=\mathbb{R}\ ,\ D_g=\mathbb{R} \Rightarrow f(x)=g(x)$

ج) $f(x)=\dfrac{4\sin^2x-4\sin x}{2\sin x-3}\ ,\ g(x)=2\sin x \rightsquigarrow D_f=\mathbb{R},\ D_g=\mathbb{R}\ ,\ f(x)=\dfrac{2\sin x(2\sin x-3)}{2\sin x-3}=2\sin x \Rightarrow f(x)=g(x)$

د) $f(x)=\dfrac{x}{|x|}\ ,\ g(x)=\dfrac{|x|}{x} \rightsquigarrow f(x)=\pm 1\ ,\ g(x)=\pm 1\ ,\ D_f=D_g=\mathbb{R}-\{0\} \Rightarrow f(x)=g(x)$

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الف) $f(x)=\left[\dfrac{x^2}{x^2+1}\right]\ ,\ g(x)=[x-[x]] \rightsquigarrow f(x)=0\ \left(0\leq\dfrac{x^2}{x^2+1}<1\right)\ ,\ g(x)=0\ ,\ D_f=D_g=\mathbb{R} \Rightarrow f(x)=g(x)$

ب) $f(x)=\dfrac{1}{[2x]}\ ,\ g(x)=\dfrac{1}{2[x]} \rightsquigarrow D_f=[0,\frac{1}{2})\ ,\ D_g=\mathbb{R}-[0,1)\ ,\ \dfrac{1}{[2x]}\neq\dfrac{1}{2[x]} \Rightarrow f(x)\neq g(x)$

ج) $f(x)=\dfrac{1}{\sqrt{x-|x|}}\ ,\ g(x)=\dfrac{1}{\sqrt{|x|-x}} \rightsquigarrow x-|x|>0 \rightsquigarrow D_f=\varnothing\ ,\ |x|-x>0 \rightsquigarrow x<0 \rightsquigarrow D_g=(-\infty,0) \Rightarrow f(x)\neq g(x)$

د) $f(x)=\begin{cases}\dfrac{x^3-x}{x-1} & ;x\neq 1\\ 2 & ;x=1\end{cases}\ ,\ g(x)=x^2+x \rightsquigarrow \dfrac{x^3-x}{x-1}=\dfrac{x(x-1)(x+1)}{x-1}=x^2+x \Rightarrow f(x)=g(x)$

$D_f=D_g=\mathbb{R}$ ← ☆ تنها عددی که ضابطه اول را تعریف نشده می‌کند، شامل تابع نیست

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$\begin{cases}f(x)+g(x)=x^2+x+1\\ f(x)-g(x)=x^2-x+1\end{cases} \rightsquigarrow 2f(x)=2x^2+2 \rightsquigarrow \boxed{f(x)=x^2+1}\ ,\ \boxed{g(x)=x}$

$f^2(x)-g^2(x)=(x^2+1)^2-x^2=x^4+1+2x^2-x^2=\boxed{x^4+x^2+1}$

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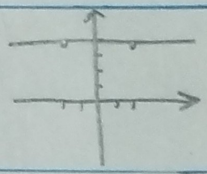
$f(x)=x-\sqrt{x^2-4} \rightsquigarrow x^2-4\geq 0 \rightsquigarrow x\geq 2,\ x\leq -2$

$g(x)=x+\sqrt{x^2-4} \rightsquigarrow x\geq 2,\ x\leq -2$

$f\times g=(x-\sqrt{x^2-4})(x+\sqrt{x^2-4})=4$



۵

$\dfrac{ax+2}{x^2-mx+n}=\dfrac{x-b}{2x^2-3x-5} \rightsquigarrow \dfrac{2ax+4}{2x^2-2mx+2n}=\dfrac{x-b}{2x^2-3x-5} \rightsquigarrow 2a=1 \Rightarrow \boxed{a=\frac{1}{2}}\ ,\ -b=4 \rightsquigarrow \boxed{b=-4}$

$2m=3 \Rightarrow \boxed{m=\frac{3}{2}}\ ,\ 2n=-5 \rightsquigarrow \boxed{n=\frac{-5}{2}}$

$am-bn=\frac{3}{4}-10=\frac{-37}{4}$

۶

$8x+b\neq 0 \rightsquigarrow x\neq\dfrac{-b}{8} \rightsquigarrow a=\dfrac{-b}{8}\ ,\ \dfrac{bx+2}{8x+b}=c \rightsquigarrow bx+2=8cx+bc \rightsquigarrow b=8c\ ,\ bc=2 \rightsquigarrow 8c^2=2 \rightsquigarrow c=\pm\dfrac{1}{2}$

$\Rightarrow b=\pm 4\ ,\ a=\pm\dfrac{1}{2} \rightsquigarrow \dfrac{ab}{c}=\pm 4$

۷

$f=\{(-1,2),(2,4),(3,-2),(0,\frac{1}{2})\}$

$g=\{(2,4),(3,7),(-1,-4),(5,0)\}$

$\dfrac{2g}{f+g}=\{(-1,-2),(2,1),(3,3)\}$

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$f=\{(2,a),(-3,3a-2b),(c,5),(2,d)\}$

$g=\{(2,-1),(-3,1),(a-3b,5)\}$

$a=(-1)\ ,\ d=(-1)$

$b=(-2)$

$c=5$

$d+c=4$

۹

$b=0 \rightsquigarrow 1-4m=0 \rightsquigarrow m=\dfrac{1}{4} \rightsquigarrow a=\dfrac{-1}{-2}=\dfrac{1}{2} \Rightarrow a+b=\dfrac{1}{2}+0=\dfrac{1}{2}$

۱۰

$\dfrac{2x+a}{x^2+4x+b}=\dfrac{2}{x-c} \rightsquigarrow 2x^2-2xc+ax-ac=2x^2+12x+2b \rightsquigarrow -2c+a=12\ ,\ -ac=2b$

$\dfrac{2(x+\frac{a}{2})}{(x+\frac{a}{2})(x-c)}=\dfrac{2}{x-c} \rightsquigarrow c=\dfrac{-a}{2} \Rightarrow a+a=12 \rightsquigarrow a=6\ ,\ c=(-3)\ ,\ b=9 \rightsquigarrow a+b+c=12$