

الف) $f(x) = \sqrt{x+1}$, $g(x) = x \rightarrow x+1 \geq 0 \rightarrow x \geq -1 \Rightarrow D_f = [-1, +\infty)$, $D_g = \mathbb{R} \Rightarrow f(x) \neq g(x)$

ب) $f(x) = \frac{(x+1)(x+1)+x+1}{x^2+2x+1}$, $g(x) = 1 \rightarrow f(x) = \frac{x^2+2x+1}{x^2+2x+1} = 1$, $D_f = \mathbb{R}$, $D_g = \mathbb{R} \Rightarrow f(x) = g(x)$

ج) $f(x) = \frac{r \sin x - r \sin x}{r \sin x - r} \rightarrow g(x) = r \sin x \rightarrow D_f = \mathbb{R}$, $D_g = \mathbb{R}$, $f(x) = \frac{r \sin x (r \sin x - r)}{r \sin x - r} = r \sin x \Rightarrow f(x) = g(x)$

د) $f(x) = \frac{x}{|x|}$, $g(x) = \frac{|x|}{x} \rightarrow f(x) = \pm 1$, $g(x) = \pm 1$, $D_f = D_g = \mathbb{R} - \{0\} \Rightarrow f(x) = g(x)$

الف) $f(x) = \left[\frac{x^r}{x^r+1} \right]$, $g(x) = [x - [x]] \rightarrow f(x) = 0$ ($0 \leq \frac{x^r}{x^r+1} < 1$), $g(x) = 0$, $D_f = D_g = \mathbb{R} \rightarrow f(x) = g(x)$

ب) $f(x) = \frac{1}{[x]}$, $g(x) = \frac{1}{r[x]}$ $\rightarrow D_f = [0, \frac{1}{r})$, $D_g = \mathbb{R} - [0, 1)$, $\frac{1}{[x]} \neq \frac{1}{r[x]} \rightarrow f(x) \neq g(x)$

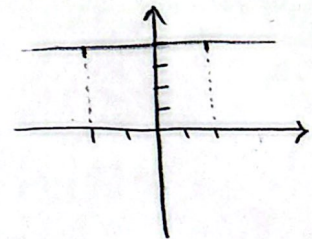
ج) $f(x) = \frac{1}{\sqrt{x-|x|}}$, $g(x) = \frac{1}{\sqrt{|x|-x}} \rightarrow x-|x| > 0 \rightarrow D_f = \emptyset$, $|x|-x > 0 \rightarrow x < 0 \rightarrow D_g = (-\infty, 0) \rightarrow f(x) \neq g(x)$

د) $f(x) = \begin{cases} \frac{x^r-x}{x-1} & ; x \neq 1 \\ r & ; x = 1 \end{cases}$, $g(x) = x^r + x \rightarrow \frac{x^r-x}{x-1} = \frac{x(x-1)(x+1)}{x-1} = x^r+x \rightarrow f(x) = g(x)$
 $D_f = D_g = \mathbb{R}$

* تنها عددی که ضابطه اول را تعریف کرده $\frac{1}{x}$ شامل تابع نیست.

$\begin{cases} f(x) + g(x) = x^r + x + 1 \\ f(x) - g(x) = x^r - x + 1 \end{cases} \Rightarrow r f(x) = r x^r + r \rightarrow f(x) = \frac{x^r+1}{r}$, $f(x) - g(x) = (x^r+1)^r - x^r = \frac{x^r+1+r x^r - x^r}{x^r+x^r+1} = \frac{r x^r+1}{x^r+x^r+1}$

$f(x) = x - \sqrt{x^r - r} \rightarrow x^r - r \geq 0 \rightarrow x \geq r, x \leq -r$
 $g(x) = x + \sqrt{x^r - r} \rightarrow x \geq r, x \leq -r$
 $f \times g = (x - \sqrt{x^r - r})(x + \sqrt{x^r - r}) = r$



$\frac{ax+r}{x^r-mx+n} = \frac{x-b}{r x^r - r m x + r n} \rightarrow \frac{r a x + r}{r x^r - r m x + r n} = \frac{x-b}{r x^r - r m x + r n} \rightarrow r a = 1 \rightarrow a = \frac{1}{r}$, $-b = r \rightarrow b = -r$
 $r m = r \rightarrow m = \frac{r}{r}$, $r n = -r \rightarrow n = -\frac{r}{r}$
 $a m - b n = \frac{1}{r} \cdot \frac{r}{r} - (-r) \cdot (-\frac{r}{r}) = \frac{1}{r} - 1 = -\frac{r-1}{r}$

$$\wedge x+b \neq 0 \leadsto x \neq \frac{-b}{\wedge} \leadsto a = \frac{-b}{\wedge}, \frac{bx+r}{\wedge x+b} = c \leadsto bx+r = \wedge cx + bc \leadsto b = \wedge c, bc=r$$

$$\star \wedge c r = r \leadsto c = \pm \frac{1}{r} / b = \pm r / a = \pm \frac{1}{r}$$

$$\boxed{\frac{ab}{c} = \pm r}$$

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$$\left. \begin{aligned} f &= \{(-1, r), (r, r), (r, -r), (0, \frac{1}{r})\} \\ g &= \{(r, r), (r, r), (-1, -r), (\wedge, 0)\} \end{aligned} \right\} \frac{rg}{f+g} = \{(-1, r), (r, 1), (r, r)\}$$

V

$$\left. \begin{aligned} f &= \{(r, a), (-r, ra - rb), (c, \omega), (r, d)\} \\ g &= \{(r, -1), (-r, 1), (a - rb, \omega)\} \end{aligned} \right\} \begin{aligned} a &= (-1) \\ b &= (-r) \\ c &= \omega \\ d &= (-1) \end{aligned} \leadsto d+c = (-1) + \omega = r$$

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$$b=0 \leadsto 1-rm=0 \leadsto m = \frac{1}{r} \leadsto a = \frac{1}{r} \leadsto a+b = \frac{1}{r} + 0 = \boxed{\frac{1}{r}}$$

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$$\frac{rx+a}{x^r+rx+b} = \frac{r}{x-c} \leadsto rx^r - rx^r c + ax - ac = rx^r + rx + rb \leadsto -rc + a = r, -ac = rb$$

$$\frac{r(x+\frac{a}{r})}{(x+\frac{a}{r})(x-c)} = \frac{r}{x-c} \leadsto c = \frac{-a}{r} \leadsto a+a=r \rightarrow a=r, c=(-r), b=r \leadsto a+b+c=r$$

$$\boxed{a+b+c=r}$$