

الف) $f(x) = \sqrt{x+1}$, $x \geq 0$, $g(x) = x \rightarrow \mathbb{R}$ برابر نیستند $\times$

ب) $f(x) = \frac{(x+3)(x+1)+x+1}{x^2+5x+6}$ $\rightarrow x^2+5x+6 \neq 0$ $g(x) = 1$ با هم برابر اند $\checkmark$
 $f(x) = \frac{x}{x} = 1$ $\rightarrow x \geq 0$ $\sqrt{x^2-2x}$

ج) $f(x) = \frac{x \sin x - 9x}{x \sin x - 3}$ $\rightarrow x \sin x - 3 \neq 0$ $x \sin x \neq 3$ $\sin x \neq \frac{3}{x}$ $\checkmark$
 $g(x) = x \sin x$

د) $f(x) = \frac{x}{|x|} \rightarrow \mathbb{R} - \{0\}$, $g(x) = \frac{|x|}{x} \rightarrow \mathbb{R} - \{0\}$ با هم برابرند $\checkmark$

الف) $f(x) = \left[\frac{x^2}{x^2+1} \right]$ و $g(x) = \left[x - \left[x \right] \right]$ $f(x) = g(x)$ برابرند $\checkmark$
 به ازای هر عدد صحیح x و y عدد صحیح

ب) $f(x) = \frac{1}{\lfloor x \rfloor}$, $g(x) = \frac{1}{\lceil x \rceil}$ $f(x) \neq g(x) \rightarrow$ مخالفند $\times$
 به طور کلی $\lfloor x \rfloor$ با $\lceil x \rceil$ از آنجا که برای اعداد صحیح برابرند و در غیر این صورت متفاوتند

ج) $f(x) = \frac{1}{\sqrt{x^2-1}}$, $g(x) = \frac{1}{\sqrt{|x|-1}}$ $|x| \geq 0 \rightarrow |x| = x$ $x - |x| = 0 \rightarrow f(x) = \frac{1}{\sqrt{0}}$ $f(x) = g(x)$ برابر نیستند $\times$
 در واقع $f(x)$ تعریف ناپذیر است $\rightarrow$ هر نقطه‌ای $x=1$ برابر در مخرج قرار دارد $f(1) = 2$ و $g(1) = 2$

د) $f(x) = \begin{cases} \frac{x^2-x}{x-1} & ; x \neq 1 \\ 2 & ; x=1 \end{cases}$, $g(x) = x^2+x$ $f(x) = g(x)$ $\checkmark$

$f(x) + g(x) = x^2+x+1$, $f(x) - g(x) = x^2-x+1$

$f^2(x) - g^2(x)$ ؟ طبقه $\checkmark$
 $f^2(x) = (x^2-x+1) + (x^2+x+1) = 2x^2+2 \rightarrow f(x) = x^2+1$

$g(x) = (x^2+x+1) - f(x) = (x^2+x+1) - (x^2+1) = x$

$f^2(x) - g^2(x) = (x^2+1)^2 - (x^2) = x^4+1+2x^2-x^2 = x^4+1+x^2$

$f(x) = x - \sqrt{x^2-4}$, $g(x) = x + \sqrt{x^2-4}$ $f \times g = ?$

$f(x)g(x) = (x - \sqrt{x^2-4})(x + \sqrt{x^2-4}) = x^2 - (x^2-4) = 4$ $\sqrt{x^2-4} \rightarrow 0 \rightarrow (-\infty, -2] \cup [2, +\infty)$

$(f \cdot g)(x) = 4$ $x = \pm 2$ رادیکال صفر است و دو تابع متضاد شش‌پایه می‌دهد $\checkmark$
 $y = 4$ $x = 2$ و $x = -2$ $\Rightarrow |x| \geq 2$ و $|x| < 2$

$f(x) = \frac{ax+b}{x^2-mx+n}$, $g(x) = \frac{x-b}{x^2-px-d}$ $\times \frac{1}{x} \rightarrow \frac{1}{x}(x-b)$

$x^2 - \frac{p}{x}x - \frac{d}{x} = x^2 - mx + n \rightarrow n = \frac{-d}{x}$, $m = \frac{p}{x}$, $\left(\frac{1}{x}\right)x - \frac{1}{x}b = ax + 2$

$\rightarrow a = \frac{1}{x}$, $-\frac{1}{x}b = 2 \rightarrow b = -2x$

$am - bn = \left(\frac{1}{x} \times \frac{p}{x}\right) - \left(-2x \times \frac{d}{x}\right) = \frac{-4d}{x}$

$$f(x) = \frac{bx+r}{1x+b}, g(a) = c \quad Dg = R - \{a\}$$

$$\frac{ab}{c} = ?$$

$$\hookrightarrow 1x+b \Rightarrow x \neq -\frac{b}{1} \Rightarrow a \neq -\frac{b}{1}$$

$$bx+r = c(1x+b) \rightarrow \{b-1c, r-bc\}, b^r=r, b=\pm r$$

$$\frac{ab}{c} = \frac{(-\frac{1}{r}) \times r}{\frac{1}{r}} = (-r) \stackrel{!}{=} \frac{ab}{c} = \frac{(\frac{1}{r} \times (-r))}{(-\frac{1}{r})} = (-r)$$

$$a = -\frac{r}{1} = -\frac{1}{r} : b = r \text{ ok}$$

$$a = -\frac{r}{1} = \frac{1}{r} : b = -r \text{ ok}$$

$$c = \frac{b}{1} = \pm \frac{1}{r}$$

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$$f-1 = \{(-1, 4), (1, 4), (3, -2), (5, 0)\}, g = \{(1, 4), (1, 4), (-1, -4), (1, 0)\}$$

$$f = \{(-1, 4), (1, 4), (3, -2), (5, \frac{1}{4})\} \quad g = \{(1, 1), (3, 1), (-1, -1), (1, 0)\}$$

$$f+g = \{(1, 1), (3, 4), (-1, -2)\} \quad 1. \quad \text{ok } \{4, 3, 1\}$$

$$\frac{fg}{f+g} = \{(1, \frac{1}{1}), (3, \frac{1}{4}), (-1, -\frac{1}{2})\} = \{(1, 1), (3, 4), (-1, -2)\}$$

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$$f = \{(1, a), (-3, 3a-2b), (c, a), (1, d)\} \quad g = \{(1, -1), (-1, 1), (a-1, b), (a)\}$$

$$(1, a), (1, d) = (1, -1) \rightarrow a = d = -1$$

$$3a-2b = 1 \rightarrow (3 \times -1) - 2b = 1 \rightarrow b = -2$$

$$a-3b = c \rightarrow -1 - (3 \times -2) = c \rightarrow c = 5$$

$$d+c = 5-1 = 4$$

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$$f(a) = \sqrt{-ax^2+bx-\frac{m}{x}}, g(a) = \{(a, b)\}$$

$$f(a) = b =$$

$$\Delta = 1^2 - 4 \times (-1) \times (-m) = 0 \rightarrow m = \frac{1}{4}$$

$$a+b = \frac{1}{4} + 0 = \frac{1}{4}$$

$$-a^2+a-\frac{1}{4} = b^2 \rightarrow -(a-\frac{1}{2})^2 = b^2 \rightarrow b=0$$

$$a-\frac{1}{2} = 0 \rightarrow a = \frac{1}{2}$$

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$$f(x) = \frac{rx+a}{x^2+rx+b}, g(a) = \frac{r}{x-c} \xrightarrow{x \rightarrow c} \frac{rx}{x^2-cx}$$

$$\hookrightarrow \frac{rx+a}{x^2+rx+b} = \frac{rx}{x^2-cx} \rightarrow rx+a=rx \rightarrow a=0$$

$$x^2+rx+b = x^2-cx \rightarrow b=0, c=-r$$

$$a+b+c = 0+0+(-r) = -r$$

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