

نام و نام خانوادگی: زهرا دهقانی

کلاس: دهم ریاضی

الف) $f(x) = \sqrt{x|x|}$, $g(x) = x \xrightarrow{x \geq 0} f(x) = \sqrt{x^2} \rightarrow f(x) = |x| = x \xrightarrow{x \geq 0} f(x) = g(x)$

ب) $f(x) = \frac{(x^2+3)(x+1)+x+5}{x^2+5x+6}$, $g(x) = 1 \Rightarrow \frac{x^2+x+3x+3+x+5}{x^2+5x+6} = \frac{x^2+5x+8}{x^2+5x+6}$
 $\Rightarrow \frac{x^2+5x+6}{x^2+5x+6} = 1 \xrightarrow{x \geq 0} f(x) = g(x)$

ج) $f(x) = \frac{x \sin x - 4 \sin x}{x \sin x - 4}$, $g(x) = x \sin x \rightarrow \frac{x \sin x - 4 \sin x}{x \sin x - 4} \xrightarrow{x \sin x \neq 4} \frac{x \sin x - 4 \sin x}{x \sin x - 4} = \frac{x \sin x - 4 \sin x}{x \sin x - 4} = 1$

د) $f(x) = \frac{x}{|x|}$, $g(x) = \frac{|x|}{x} \xrightarrow{x < 0} f(x) = \frac{x}{-x} = -1$, $g(x) = \frac{-x}{x} = -1 \Rightarrow f(x) = g(x) = -1$
 $\xrightarrow{x > 0} f(x) = \frac{x}{x} = 1$, $g(x) = \frac{x}{x} = 1 \Rightarrow f(x) = g(x) = 1$

هـ) $f(x) = \left[\frac{x^2}{x^2+1} \right]$, $g(x) = [x - [x]] \rightarrow f(x) = g(x) = 0$
 $x \in \mathbb{R} \rightarrow 0$
 $x^2+1 \neq 0 \rightarrow x^2 \neq -1$

و) $f(x) = \frac{1}{\sqrt{x-1}}$, $g(x) = \frac{1}{\sqrt{|x|-1}}$
 $\xrightarrow{x > 1} f(x) = \frac{1}{\sqrt{x-1}}$, $g(x) = \frac{1}{\sqrt{x-1}} \Rightarrow f(x) = g(x)$
 $\xrightarrow{x < 1} f(x) = \frac{1}{\sqrt{1-x}}$, $g(x) = \frac{1}{\sqrt{1-x}} \Rightarrow f(x) = g(x)$
 $\xrightarrow{x=1} f(x) = \frac{1}{\sqrt{0}}$, $g(x) = \frac{1}{\sqrt{0}} \Rightarrow f(x) = g(x)$

ز) $f(x) + g(x) = x^2 + x + 1$, $f(x) - g(x) = x^2 - x + 1$
 $\Rightarrow \begin{cases} f(x) + g(x) = x^2 + x + 1 \\ f(x) - g(x) = x^2 - x + 1 \end{cases} \Rightarrow \begin{cases} 2f(x) = 2x^2 + 2 \Rightarrow f(x) = x^2 + 1 \\ 2g(x) = 2x \Rightarrow g(x) = x \end{cases}$

$f(x) - g(x) = (f(x) + g(x))(f(x) - g(x)) = (x^2 + 1 - x)(x^2 + 1 + x)$
 $= x^4 + x^2 + 1 - x^3 - x - x^2 - 1 = x^4 - x^3 - x$
 جواب نهایی

$f(x) \times g(x) = (x - \sqrt{x^2 - 4})(x + \sqrt{x^2 - 4})$

$a = x$, $b = \sqrt{x^2 - 4}$

$(a-b)(a+b) = a^2 - b^2 \rightarrow x^2 - (\sqrt{x^2 - 4})^2 = 4$

$\frac{x-b}{x^2-3x-4} = \frac{ax+2}{x^2-mx+n} \xrightarrow{\text{مخرج مساوی}} \begin{cases} x^2-3x-4 = x^2-mx+n \\ ax+2 = x-mx \end{cases} \Rightarrow \begin{cases} -3 = -m \\ -4 = n \end{cases} \Rightarrow \begin{cases} m = 3 \\ n = -4 \end{cases}$

$am - bn = (3 \times \frac{3}{4}) - (-1 \times -\frac{4}{4}) = \frac{9}{4} - 1 = \frac{5}{4}$
 $b - b = 1 \Rightarrow b = -1$

$$f(x) = \frac{bx + r}{ax + b}$$

$$ax + b \rightarrow ax + b \neq 0 \rightarrow x \neq -\frac{b}{a} \implies D_f = \mathbb{R} - \left\{-\frac{b}{a}\right\}$$

$$D_f = D_g \rightarrow -\frac{b}{a} = -\frac{r}{a} \rightarrow b = -a^r$$

$$f(x) = g(x) = c = \frac{bx + r}{ax + b} \rightarrow \frac{-a^r x + r}{ax - a^r} = c \rightarrow -a^r x + r = cax - ca^r$$

$$c = -a^r \rightarrow -(-a^r)x + r = a^r x - a^r \rightarrow r = -a^r \rightarrow a = \sqrt[r]{r} \rightarrow \frac{ab}{c} = \sqrt[r]{r}$$

$$D_f = \{3, 4, 0, -1\} \rightarrow D_f \cap D_g = 3$$

$$D_g = \{1, 3, 4, -1\}$$

$$f = \{(3, -1), (4, 1), (-1, 3), (0, 0)\}$$

$$g = \{(1, 1), (3, 4), (-1, -1), (4, 0)\}$$

$$f(3) = -1$$

$$g(3) = 4$$

$$\frac{fg}{f+g} = \frac{3 \times 4}{4-1} = \frac{12}{3}$$

$$x=3$$

$$3a - 4b = 1 \leftarrow f(-3, 3a - 4b), g(-3, 1)$$

$$d + c = a - 1 = 1$$

$$3(-1) - 4b = 1 \rightarrow -3 - 4b = 1 \rightarrow -4 = 4b \rightarrow b = -1$$

$$g = (a - 3b, d), f = (c, d)$$

$$c = -1 - 3(-1) = 2$$

$$-x^y + x - m = 0 \rightarrow -(-x^y + x - m) = 0 \rightarrow x^y - x + m = 0$$

$$\Delta = 1 - 4m = 0 \rightarrow m = \frac{1}{4}$$

$$x = \frac{1}{2} \rightarrow -x^y + x - m = 0$$

$$f\left(\frac{1}{2}\right) = 0$$

$$\frac{yx + a}{x^y + 4x + b} = \frac{y}{x - c} \rightarrow (yx + a)(x - c) = y(x^y + 4x + b)$$

$$\rightarrow yx^y - yxc + ax - ac = yx^y + 4yx + yb$$

$$\rightarrow -yxc + ax - ac = 4yx + yb$$

$$\rightarrow x(-yc + a) - ac = 4yx + yb$$

$$\rightarrow -yc + a = 4y$$

$$a = 4y + yc$$

$$-ac = yb \rightarrow b = -\frac{ac}{y}$$

$$b = -\frac{(4y + yc)c}{y} = -4c - c^2$$

$$a + b + c$$

$$\Rightarrow (4y + yc) + (-4c - c^2) + c$$

$$= 4y - c^2 - 3c$$