

الف) $f(x) = \sqrt{x|x|}$, $g(x) = x \rightarrow \begin{cases} D_g = \mathbb{R} \\ D_f = [0, +\infty) \end{cases} \rightarrow \text{تابع با هم برابر نیست}$

①

② ✓ $x|x| \geq 0$

ب) $f(x) = \frac{(x+3)(x+1)+x+4}{x^2+4x+7}$, $g(x) = 1 \rightarrow \begin{cases} D_g = \mathbb{R} \\ D_f = \mathbb{R} \end{cases} \rightarrow D_g = D_f$

$\Delta = b^2 - 4ac = 16 - 28 = -12$ $\rightarrow$ ریشه ندارد

$g(x) = 1$

$f(x) = \frac{x^2+4x+7+x+4}{x^2+4x+7} = \frac{x^2+5x+11}{x^2+4x+7} = 1$

$\downarrow$

$f(x) = g(x)$

تابع با هم برابرند

ج) $f(x) = \frac{4\sin^2 x - 4\sin x}{4\sin x - 4}$, $g(x) = 4\sin x \rightarrow \begin{cases} D_g = \mathbb{R} \\ D_f = \mathbb{R} \end{cases} \rightarrow D_g = D_f$

$4\sin x \neq 4 \rightarrow \sin x \neq 1$ $\rightarrow$ ریشه ندارد

$g(x) = 4\sin x$

$f(x) = \frac{4\sin x (4\sin x - 4)}{4\sin x - 4} = 4\sin x$

$\downarrow$

$f(x) = g(x)$

تابع با هم برابرند

د) $f(x) = \frac{x}{|x|}$, $g(x) = \frac{|x|}{x} \rightarrow \begin{cases} D_g = \mathbb{R} - \{0\} \\ D_f = \mathbb{R} - \{0\} \end{cases} \rightarrow D_g = D_f$

$g(x) = \begin{cases} x > 0 \rightarrow \frac{x}{x} = 1 \\ x < 0 \rightarrow \frac{-x}{-x} = -1 \end{cases}$

$f(x) = \begin{cases} x > 0 \rightarrow \frac{x}{x} = 1 \\ x < 0 \rightarrow \frac{-x}{x} = -1 \end{cases} \rightarrow f(x) = g(x)$

تابع با هم برابرند

الف) $f(x) = \left[\frac{x^2}{x^2+1} \right]$, $g(x) = [x - [x]] \rightarrow \begin{cases} D_g = \mathbb{R} \\ D_f = \mathbb{R} \end{cases} \rightarrow D_g = D_f$

$\downarrow$ $\frac{x^2}{x^2+1}$ $\rightarrow$ همیشه صفر است

$g(x) = 0$

$f(x) = \left[\frac{x^2+1-1}{x^2+1} \right] = \left[1 - \frac{1}{x^2+1} \right] = [1^-] = 0$

$\downarrow$

$f(x) = g(x)$

تابع با هم برابرند

②

ب) $f(x) = \frac{1}{[2x]}$, $g(x) = \frac{1}{2[x]} \rightarrow \begin{cases} D_g = 2[x] \neq 0 \rightarrow [x] \neq 0 \rightarrow x < 1 \rightarrow D_g = (0, 1) \\ D_f = [2x] \neq 0 \rightarrow 0 < 2x < 1 \rightarrow 0 < x < \frac{1}{2} \rightarrow D_f = (0, \frac{1}{2}) \end{cases} \rightarrow D_g \neq D_f$

تابع با هم برابر نیست.

$$2) f(x) = \frac{1}{\sqrt{x-|x|}}, g(x) = \frac{1}{\sqrt{|x|-x}} \rightarrow \begin{cases} D_g = |x|-x > 0 \rightarrow |x| > x \rightarrow D_g = (-\infty, 0) \cup (0, +\infty) \\ D_f = x-|x| > 0 \rightarrow x > |x| \rightarrow D_f = \emptyset \end{cases}$$

تابع با هم برابر نیست

$$D_g \neq D_f$$

$$3) f(x) = \begin{cases} \frac{x^r - x}{x-1} & : x \neq 1 \\ \gamma & : x = 1 \end{cases}, g(x) = x^r + x \rightarrow \begin{cases} g(x) = x^r + x \\ f(x) = \begin{cases} x=1 \rightarrow 1+1=2 \checkmark \\ x \neq 1 \rightarrow \frac{x(x^r-1)}{x-1} = \frac{x(x-1)(x^{r-1}+x^{r-2}+\dots+1)}{x-1} = x(x^{r-1}+x^{r-2}+\dots+1) \end{cases} \end{cases}$$

$D_g = D_f$

تابع با هم برابر است

$$f(x) + g(x) = x^r + x + 1 \rightarrow f(x) = \frac{(x^r + x + 1) + (x^r - x + 1)}{2} = \frac{2x^r + 2}{2} = x^r + 1$$

$$f(x) - g(x) = x^r - x + 1 \rightarrow g(x) = \frac{(x^r + x + 1) - (x^r - x + 1)}{2} = \frac{2x}{2} = x$$

$$\begin{cases} f'(x) = (x^r + 1)' = x^{r-1} + 1 + \gamma x^{r-1} \\ g'(x) = x^r \end{cases} \rightarrow f'(x) - g'(x) = x^{r-1} + 1 + \gamma x^{r-1} - x^{r-1} = x^{r-1} + 1$$

$$f(x) = x - \sqrt{x^r - r}, g(x) = x + \sqrt{x^r - r} \rightarrow f \cdot g = (x - \sqrt{x^r - r})(x + \sqrt{x^r - r}) = x^2 - (x^r - r) = x^2 - x^r + r = r$$

$x^r - r > 0 \rightarrow x^r > r \rightarrow -1 \leq x \leq 1$

$$f(x) = \frac{ax+r}{x^r-mx+n}, g(x) = \frac{x-b}{kx^r-lx-d} \rightarrow x-b = k(ax+r) \rightarrow kax + rk = x - b \rightarrow \begin{cases} ka = 1 \rightarrow a = \frac{1}{k} \\ -b = rk \rightarrow b = -rk \end{cases}$$

$$kx^r - lx - d = k(ax^r - mx + n) \rightarrow \begin{cases} m = \frac{r}{k} \\ n = -\frac{d}{k} \end{cases}$$

تابع با هم برابر است

$$am - bn \rightarrow \left(\frac{1}{k} \cdot \frac{r}{k} \right) - \left(-rk - \frac{d}{k} \right) = \frac{r}{k} + rk + \frac{d}{k} = \frac{r^2 + k^2 d + r^2}{k^2}$$

$$f(x) = \frac{bx+c}{ax+b} \rightarrow D_f = R - \left\{ -\frac{b}{a} \right\}$$

(4)

$$\frac{ab}{c} = \frac{ab}{a} = b \rightarrow b = -1a$$

معادله خطی در دو مجهول است و می توانیم با استفاده از آن

$$f(x) = c \rightarrow D_f = D_g \rightarrow -\frac{b}{a} = a$$

$$D_g = R - \{a\} \rightarrow a = c$$

مقادیر

$$f^{-1} = \{(-1, 2), (1, 2), (2, -1), (0, \frac{1}{2})\} \rightarrow f = \{(-1, 2), (1, 2), (2, -1), (0, \frac{1}{2})\}$$

(5)

$$g = \{(1, 2), (2, 2), (-1, -2), (0, 0)\}$$

$$\frac{f_g}{f+g} = \left\{ \left(-1, \frac{-1}{-1} \right), \left(1, \frac{1}{1} \right), \left(2, \frac{1}{2} \right) \right\}$$

$$\frac{f_g}{f+g} \text{ بردار } \rightarrow R = \{+1, 1, 2\}$$

$$f = \{(1, a), (-1, 2a-2b), (2, a), (1, d)\}$$

$$a = -1 \quad a = d \rightarrow d = -1$$

(1)

$$2a-2b=1 \rightarrow -1-2b=1 \rightarrow \frac{-2b}{-2} = \frac{2}{-2} \Rightarrow b = -1$$

$$g = \{(1, -1), (-1, 1), (a-2b, a)\}$$

$$a-2b = c \rightarrow -1+2 = \boxed{1}$$

$$2ab+d+c = -1+d = \boxed{1}$$

$$f(x) = \sqrt{-x^2+x-m}$$

$$\Delta = 0$$

$$\Delta = b^2 - 4ac \rightarrow 1 - 4m = 0 \rightarrow m = \frac{1}{4}$$

$$\rightarrow f(x) = \sqrt{-x^2+x-\frac{1}{4}}$$

(9)

$$g(x) = \{(a, b)\}$$

$$a = \frac{1}{4} \quad b = 0$$

$$f(x) = \sqrt{-(x-\frac{1}{4})^2}$$

$$a+b = \frac{1}{4} + 0 = \boxed{\frac{1}{4}}$$

$$= \left\{ \left(\frac{1}{4}, 0 \right) \right\}$$

$$f(x) = \frac{2x+a}{x^2+2x+b}$$

$$f(x) = \frac{2x+a}{(x+1)^2} = \frac{2}{x+1}$$

$$(x+1)^2 = x^2 + 2x + 1 = \boxed{b=1}$$

(10)

$$f(x) = \frac{2}{x-1}$$

$$g(x) = \frac{2}{x-1} \rightarrow \boxed{C = -2}$$

$$\frac{2(x+1)}{(x+1)^2} = \frac{2}{x+1} \rightarrow \boxed{a=4}$$

$$a+b+c = 4+1-2 = \boxed{3}$$