

الف) $f(x) = \sqrt{x|x|}$, $g(x) = x \rightarrow \begin{cases} D_g = \mathbb{R} \\ D_f = [0; +\infty) \end{cases} \rightarrow \text{تابع با هم برابر نیست}$ ✓

⊖
⊕ $\checkmark$ $x|x| \geq 0$

ب) $f(x) = \frac{(x+3)(x+1)+x+4}{x^2+4x+7}$, $g(x) = 1 \rightarrow \begin{cases} D_g = \mathbb{R} \\ D_f = \mathbb{R} \end{cases} \rightarrow D_g = D_f$

$\Delta = b^2 - 4ac = 16 - 28 = -12$ $\checkmark$

$g(x) = 1$
 $f(x) = \frac{x^2+4x+7}{x^2+4x+7} = 1$
 $\downarrow$
 $f(x) = g(x)$

تابع با هم برابرند

ج) $f(x) = \frac{5\sin^2 x - 4\sin x}{5\sin x - 4}$, $g(x) = 5\sin x \rightarrow \begin{cases} D_g = \mathbb{R} \\ D_f = \mathbb{R} \end{cases} \rightarrow D_g = D_f$

$5\sin x \neq 4 \rightarrow \sin x \neq \frac{4}{5}$ $\checkmark$

$g(x) = 5\sin x$
 $f(x) = \frac{5\sin x (5\sin x - 4)}{5\sin x - 4} = 5\sin x$
 $\downarrow$
 $f(x) = g(x)$

تابع با هم برابرند

د) $f(x) = \frac{x}{|x|}$, $g(x) = \frac{|x|}{x} \rightarrow \begin{cases} D_g = \mathbb{R} - \{0\} \\ D_f = \mathbb{R} - \{0\} \end{cases} \rightarrow D_g = D_f$

$g(x) = \begin{cases} x > 0 \rightarrow \frac{x}{x} = 1 \\ x < 0 \rightarrow \frac{-x}{-x} = -1 \end{cases}$
 $f(x) = \begin{cases} x > 0 \rightarrow \frac{x}{x} = 1 \\ x < 0 \rightarrow \frac{-x}{x} = -1 \end{cases} \rightarrow f(x) = g(x)$ ✓

تابع با هم برابرند

الف) $f(x) = \left[\frac{x^2}{x^2+1} \right]$, $g(x) = [x - [x]] \rightarrow \begin{cases} D_g = \mathbb{R} \\ D_f = \mathbb{R} \end{cases} \rightarrow D_g = D_f$

$f(x) = \left[\frac{x^2+1-1}{x^2+1} \right] = \left[1 - \frac{1}{x^2+1} \right] = [1^-] = 0$
 $\downarrow$
 $f(x) = g(x)$ $\checkmark$

تابع با هم برابرند

ب) $f(x) = \frac{1}{[2x]}$, $g(x) = \frac{1}{2[x]} \rightarrow \begin{cases} D_g = 2[x] \neq 0 \rightarrow [x] \neq 0 \rightarrow x < 1 \text{ و } x \geq 0 \rightarrow D_g = [0, 1) \\ D_f = [2x] \neq 0 \rightarrow 0 < 2x < 1 \rightarrow 0 < x < \frac{1}{2} \rightarrow D_f = [0, \frac{1}{2}) \end{cases} \rightarrow D_g \neq D_f$

تابع با هم برابر نیست.

$$2) f(x) = \frac{1}{\sqrt{x-|x|}}, g(x) = \frac{1}{\sqrt{|x|-x}} \rightarrow \begin{cases} D_g = |x|-x > 0 \rightarrow |x| > x \rightarrow D_g = (-\infty, 0) \cup (0, +\infty) \\ D_f = x-|x| > 0 \rightarrow x > |x| \rightarrow D_f = \emptyset \end{cases}$$

تابع زوج

$D_g \neq D_f$

$$3) f(x) = \begin{cases} \frac{x^r - x}{x-1} & : x \neq 1 \\ \gamma & : x = 1 \end{cases}, g(x) = x^r + x \rightarrow \begin{cases} g(x) = x^r + x \\ f(x) = \begin{cases} x=1 \rightarrow 1+1=2 \\ x \neq 1 \rightarrow \frac{x(x^r-1)}{x-1} = \frac{x(x-1)(x^{r-1}+x^{r-2}+\dots+1)}{x-1} = x(x^{r-1}+x^{r-2}+\dots+1) \end{cases} \end{cases}$$

$D_g = D_f$

تابع زوج

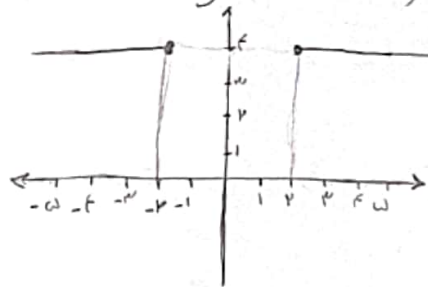
$$f(x) + g(x) = x^r + x + 1 \rightarrow f(x) = \frac{(x^r + x + 1) + (x^r - x + 1)}{2} = \frac{2x^r + 2}{2} = x^r + 1$$

$$f(x) - g(x) = x^r - x + 1 \rightarrow g(x) = \frac{(x^r + x + 1) - (x^r - x + 1)}{2} = \frac{2x}{2} = x$$

$$\begin{cases} f'(x) = (x^r + 1)' = x^r + 1 + \gamma x^r \\ g'(x) = x^r \end{cases} \rightarrow f'(x) - g'(x) = x^r + 1 + \gamma x^r - x^r = x^r + 1$$

$$f(x) = x - \sqrt{x^r - r}, g(x) = x + \sqrt{x^r - r} \rightarrow f(x)g(x) = (x - \sqrt{x^r - r})(x + \sqrt{x^r - r}) = x^2 - (x^r - r) = x^2 - x^r + r$$

$$x^r - r > 0 \rightarrow x^r > r \rightarrow -1 \leq x \leq 1$$



$$f(x) = \frac{ax+r}{x^r-mx+n}, g(x) = \frac{x-b}{kx^r-lx-d} \rightarrow x-b = k(ax+r) \rightarrow kax + rk = x - b \rightarrow \begin{cases} ka = 1 \\ -b = rk \end{cases}$$

$$kx^r - lx - d = k(ax+r) \rightarrow \begin{cases} m = \frac{r}{k} \\ n = -\frac{d}{k} \end{cases}$$

$$am - bn \rightarrow \left(\frac{1}{k} \cdot \frac{r}{k} \right) - \left(-\frac{r}{k} \cdot \frac{d}{k} \right) = \frac{r}{k^2} + \frac{rd}{k^2} = \frac{r(r+d)}{k^2}$$

$$f(x) = \frac{bx+c}{ax+b} \rightarrow D_f = R - \left\{ -\frac{b}{a} \right\}$$

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$$g(x) = c$$

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$$D_g = R - \{a\} \rightarrow a=c$$

$$ax+b \neq 0 \rightarrow x \neq -\frac{b}{a}$$

$$\frac{ab}{c} = \frac{ab}{a} = b \rightarrow b = -1a$$

توجه: هرگاه که $a=0$ و $b=0$ و $c=0$ باشد، تابع $f(x)$ به ازای هر x در R تعریف می‌شود.

توجه: اگر $a=0$ و $b=0$ و $c=0$ باشد، تابع $f(x)$ به ازای هر x در R تعریف می‌شود.

$$f^{-1} = \{(-1, 2), (1, 4), (3, -2), (0, 1)\} \rightarrow f = \{(-1, 2), (1, 4), (3, -2), (0, 1)\}$$

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$$g = \{(1, 4), (3, 6), (-1, -4), (0, 0)\}$$

$$\frac{fg}{f+g} = \left\{ \left(-1, \frac{-1}{-1} \right), \left(1, \frac{1}{1} \right), \left(3, \frac{12}{5} \right) \right\}$$

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$$\frac{fg}{f+g} \text{ بر مبنای } R = \{+1, 1, 3\}$$

$$f = \{(1, a), (-1, 2a-2b), (3, a), (1, d)\}$$

$$a=-1 \quad a=d \rightarrow d=-1$$

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$$g = \{(1, -1), (-1, 1), (a-2b, a)\}$$

$$3a-2b=1 \rightarrow -3-2b=1 \rightarrow \frac{-2b}{-2} = \frac{4}{-2} \Rightarrow b=-2$$

$$a-2b=c \rightarrow -1+4=c \Rightarrow c=3$$

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$$2ab+d+c = -1+d = 1 \Rightarrow d=2$$

$$f(x) = \sqrt{-x^2+x-m}$$

$$\Delta=0$$

$$\Delta=b^2-4ac$$

$$1-4m=0 \rightarrow m=\frac{1}{4} \rightarrow f(x) = \sqrt{-x^2+x-\frac{1}{4}}$$

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$$g(x) = \{(a, b)\}$$

$$a=\frac{1}{p} \quad b=0$$

$$f(x) = \sqrt{-(x-\frac{1}{p})^2}$$

$$a+b = \frac{1}{p} + 0 = \frac{1}{p}$$

$$= \left\{ \left(\frac{1}{p}, 0 \right) \right\}$$

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$$f(x) = \frac{px+a}{x^2+qx+b}$$

$$f(x) = \frac{px+a}{(x+1)^2} = \frac{p}{x+1}$$

$$(x+1)^2 = x^2+2x+1 = b \Rightarrow b=1$$

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$$g(x) = \frac{p}{x-c} \rightarrow c=-2$$

$$\frac{p(x+1)}{(x+1)^2} = \frac{p}{x+1} \rightarrow a=4$$

$$a+b+c = 4+9-3 = 11$$

$$a+b+c = 4+9-3 = 11$$

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