

۱ $2f(x) \geq 0$

$(Df = [-\infty, 0] \cup [2, \infty])$

	-4	-2	0	2	4
x	-	-	+	+	+
f(x)	+	+	-	-	+
2f(x)	-	-	+	+	+

۲ $\frac{-x}{f(x)} \geq 0 \Rightarrow -x \geq 0, f(x) > 0$

~~$(Df = (-\infty, 0) \cup (2, \infty))$~~

	-4	-2	0	2	4
-x	+	+	0	-	-
f(x)	-	-	0	+	+
$\frac{-x}{f(x)}$	-	-	0	-	-

~~شکل منحنی در ضمیمه است.~~

$(-\infty, 0) \cup (2, \infty) \Rightarrow -2, -1$ و $(0, 2) \Rightarrow 1, (\infty, \infty) \Rightarrow 2, 3, \dots$

۳ $f(x) - 2f(2) = x^2 - 2x + 4, f(-2) = ?$

۱) $f(2) = ? \xrightarrow{x=2} f(2) - 2f(2) = 4 - 4 + 4 = -f(2) = 2 \Rightarrow f(2) = -2$

$f(-2) \xrightarrow{x=-2} f(-2) - 2f(2) = 4 + 4 + 4 \Rightarrow f(-2) = 14 - 4 \Rightarrow f(-2) = 10$

۴ $f(x) = \begin{cases} x - \sqrt{x+4} & ; x > 2 \\ 2x+3 & ; x \leq 2 \end{cases}$

* $f(f(4)) + f(f(1)) = ?$

۱) $f(4) = 4 - \sqrt{4+4} = 4 - 2 = 2, f(1) = (2 \times 1) + 3 = 5$

۲) $f(f(4)) + f(f(1)) = f(2) + f(5) = ?$

۳) $f(2) = 2 \times 2 + 3 = 7, f(5) = 5 - \sqrt{5+4} = 5 - 3 = 2$

پس: $f(f(4)) + f(f(1)) = f(2) + f(5) = 7 + 2 = 9$

۵ $f(x) = ax^2 - bx + 2, f(x-1) - f(x) = 4x+2$

$(a-b) = ?$

① $f(x-1) = a(x-1)^2 - b(x-1) + 2 = ax^2 - 2ax + a - bx + b + 2, -f(x) = -ax^2 + bx - 2$

② $f(x-1) - f(x) = ax^2 - 2ax + a - bx + b + 2 - ax^2 + bx - 2 = -2ax + a + b$

$f(x-1) - f(x) = 4x+2$

$-2ax + a + b = 4x+2$

$\begin{cases} -2a = 4 \Rightarrow -a = 2 \Rightarrow a = -2 \\ a + b = 2 \Rightarrow -2 + b = 2 \Rightarrow b = 4 \end{cases}$

$a - b = -2 - 4 = -6$

$f(x) = \frac{x^r + x + a}{x^r + x + v}, \quad f(\sqrt{r} - r) = ?$ $f(x) = \frac{x^r + x + a}{x^r + x + v} = \frac{(x^r + x + v) - r}{x^r + x + v} = 1 - \frac{r}{x^r + x + v}$ $f(\sqrt{r} - r) = 1 - \frac{r}{(\sqrt{r} - r)^r + (\sqrt{r} - r) + v} = 1 - \frac{r}{r + r - r\sqrt{r} + \sqrt{r} - r + v} = 1 - \frac{r}{v} = 1 - \frac{1}{\frac{v}{r}} = \frac{v}{r} *$ $\begin{cases} (\sqrt{r} - r)^r = r + r - r\sqrt{r} \\ f(\sqrt{r} - r) = r + \sqrt{r} - r \end{cases}$ راه: $f(\sqrt{r} - r) = 1 - \frac{r}{v}$ است	۲ ۶
$f(x - \frac{1}{x}) = \frac{x^r + 1}{x^r}, \quad f(\frac{1}{x}) = ?$ $f(x - \frac{1}{x}) = \frac{x^r + \frac{1}{x^r}}{x^r} \Rightarrow f(x - \frac{1}{x}) = (x - \frac{1}{x})^r + r$ $\xrightarrow{\quad} f(x) = x^r + r$ $f(r) = (x^r)^r + r = a + r = \frac{11}{2}$	۲ ۷
$g(x) = \sqrt{a - x^r}, \quad a - x^r \geq 0 \Rightarrow -x^r \leq x^r \Rightarrow [-r, r]$ $f(x) = \{ (r, 0)^*, (1, -\frac{r}{\sqrt{r}})^*, (0, \frac{r}{r})^* \}$ * : شیب ا) $f_g = \{ (r, 0), (1, -\frac{r}{\sqrt{r}}), (0, \frac{r}{r}) \}$ $\hookrightarrow \frac{r}{\sqrt{r}} \times \frac{\sqrt{r}}{\sqrt{r}} = \frac{r\sqrt{r}}{r} = \sqrt{r}$ ب) $\frac{g}{f} = \{ (r, \frac{\sqrt{r}}{r}), (1, \frac{\sqrt{r}}{-r}), (0, \frac{r}{r}) \} = \{ (1, -\frac{\sqrt{r}}{r}), (0, \frac{r}{r}) \}$ $\frac{-\sqrt{r}}{r}$	۲ ۸
$f(x) = \{ (r, 1)(r, r)(-a, r)(1, -r) \}$ ا) $r f(x) = \{ (r, r)(r, 1)(-a, r)(1, -r) \}$ ✓ ب) $f(x) + 1 = \{ (r, r)(r, a)(-a, r)(1, -1) \}$ ✓ ج) $r f(x) + 1 = \{ (r, r)(r, r)(-a, r)(1, -r) \}$ ✓ $f(x) = \{ (r, 1)(r, 1)(-a, r)(1, r) \} x^r + 1$ د) $f(rx) = \{ (1, 1)(\frac{r}{r}, r)(\frac{-a}{r}, r)(\frac{1}{r}, -r) \} = \{ (1, 1)(1, a, r)(-r, a, r)(r, a, -r) \}$ ✓ * : شیب	۲ ۹
$g(x) = \{ (1, 2)^*, (r, 1)(-1, r)(-r, -r)(r, -1) \}$ $f(x) = \{ (r, r)(r, r)(1, r)(r, 1)(a, r) \}$ ا) $f - g = \{ (1, r)(r, r)(r, r) \}$ ب) $\frac{rf}{g} = \{ (1, \frac{r}{r})(r, r)(r, -r) \} = \{ (r, r)(r, -r) \}$ $rf(x) = \{ (r, r)(r, r)(1, 1)(r, r)(a, r) \}$ * : شیب	۲ ۱۰