

الف) $y = \frac{\sin x - 2}{2 \cos x + 1}$

$2 \cos x + 1 \neq 0 \rightarrow \cos x \neq -\frac{1}{2} \rightarrow \begin{matrix} \text{Unit Circle} \\ \text{Points: } (1,0), (2,0) \end{matrix} \quad D_f = \mathbb{R} - \left\{ 2k\pi + k\pi \pm \frac{2\pi}{3} \right\}$

ب) $y = \frac{\sin x + 2}{\cos x - 1}$

$\cos x - 1 \neq 0 \rightarrow \cos x \neq 1 \rightarrow \begin{matrix} \text{Unit Circle} \\ \text{Point: } (1,0) \end{matrix} \quad D_f = \mathbb{R} - \{ 2k\pi \}$

الف) $y = \frac{2 \sin x + 1}{\tan x + 1}$

$\tan x + 1 \neq 0 \rightarrow \frac{\sin}{\cos} + 1 \neq 0 \rightarrow \frac{\sin}{\cos} \neq -1 \rightarrow \begin{matrix} \text{Unit Circle} \\ \text{Points: } (1,1), (-1,-1) \end{matrix} \quad D_f = \mathbb{R} - \left\{ k\pi + \frac{\pi}{4}, k\pi - \frac{\pi}{4} \right\}$

ب) $y = \frac{\cos x + 1}{\cot x - 1}$

$\frac{\cos}{\sin} - 1 \neq 0 \rightarrow \frac{\cos}{\sin} \neq 1 \rightarrow \begin{matrix} \text{Unit Circle} \\ \text{Points: } (1,1), (-1,-1) \end{matrix} \quad D_f = \mathbb{R} - \left\{ k\pi, k\pi + \frac{\pi}{2} \right\}$

الف) $\sin y = x^2 - 2 \rightarrow -1 \leq x^2 - 2 \leq 1 \rightarrow -1 \leq x^2 \leq 3 \rightarrow |x| \leq \sqrt{3} \rightarrow D_f = [-\sqrt{3}, -1] \cup [1, \sqrt{3}]$

ب) $y = \arccos(\sqrt{x} - 2) \rightarrow \begin{cases} \sqrt{x} \geq 0 \rightarrow x \geq 0 \\ -1 \leq \sqrt{x} - 2 \leq 1 \rightarrow 1 \leq \sqrt{x} \leq 3 \rightarrow 1 \leq x \leq 9 \end{cases} \quad D_f = [1, 9]$

الف) $\cos y = |x| - 2 \rightarrow -1 \leq |x| - 2 \leq 1 \rightarrow -1 \leq |x| \leq 3 \rightarrow |x| \leq 3 \rightarrow D_f = [-3, -1] \cup [1, 3]$

ب) $y = \arcsin(x^2 + 2x + 1) \rightarrow -1 \leq x^2 + 2x + 1 \leq 1 \rightarrow \begin{cases} x^2 + 2x + 1 \leq 1 \rightarrow x^2 + 2x \leq 0 \rightarrow x(x+2) \leq 0 \rightarrow x \in [-2, 0] \\ x^2 + 2x + 1 \geq -1 \rightarrow x^2 + 2x + 2 \geq 0 \rightarrow (x+1)^2 + 1 \geq 0 \rightarrow \text{Always true} \end{cases} \rightarrow D_f = [-2, 0]$

الف) $y = \log_{x^2-2} x \rightarrow \begin{cases} x^2 - 2 > 0 \rightarrow x > \sqrt{2} \text{ or } x < -\sqrt{2} \\ x^2 - 2 \neq 1 \rightarrow x \neq \pm \sqrt{3} \end{cases} \rightarrow D_f = (-\infty, -\sqrt{3}) \cup (-\sqrt{2}, -\sqrt{3}) \cup (\sqrt{2}, \sqrt{3}) \cup (\sqrt{3}, +\infty)$

ب) $y = \log_{x^2-1} |x| \rightarrow x^2 - 1 > 0 \rightarrow x > 1 \text{ or } x < -1 \rightarrow D_f = (-\infty, -1) \cup (1, +\infty)$

الف) $y = \log_{x-2}^{x-1} \rightarrow \begin{cases} x-1 > 0 \rightarrow x > 1 \\ x-2 > 0 \rightarrow x > 2 \\ x-2 \neq 1 \rightarrow x \neq 3 \end{cases} \rightarrow D_f = (2, 3) \cup (3, +\infty)$

ب) $y = \log_{x+2}^{x-1} \rightarrow \begin{cases} x-1 > 0 \rightarrow x > 1 \\ x+2 > 0 \rightarrow x > -2 \\ x+2 \neq 1 \rightarrow x \neq -1 \end{cases} \rightarrow D_f = (-1, -2) \cup (-2, 1) \cup (1, +\infty)$

الف) $y = \log_{x^2-2x+3} \frac{x^2-2x+3}{x-2} \rightarrow \begin{cases} x^2-2x+3 > 0 \rightarrow \text{Always true} \\ x^2-2x+3 \neq 1 \rightarrow x \neq 1, 3 \end{cases} \rightarrow D_f = (1, 3) \cup (3, +\infty)$

ب) $y = \log_{x+2} \frac{x+2}{x-2} \rightarrow \begin{cases} x+2 > 0 \rightarrow x > -2 \\ x+2 \neq 1 \rightarrow x \neq -1 \end{cases} \rightarrow D_f = (-2, -1) \cup (-1, +\infty)$

الف) $y = \sqrt{r - \log_r(x-2)} \rightarrow \begin{cases} r - \log_r(x-2) \geq 0 \rightarrow \log_r(x-2) \leq r \rightarrow x-2 \leq r^r \rightarrow x \leq r^r + 2 \end{cases} \rightarrow D_f = (2, r^r + 2]$

ب) $y = \log(r \log_r x - 1) \rightarrow r \log_r x - 1 > 0 \rightarrow r \log_r x > 1 \rightarrow \log_r x > \frac{1}{r} \rightarrow x > r^{\frac{1}{r}} \rightarrow D_f = (r^{\frac{1}{r}}, +\infty)$

$$الف) y = \frac{3}{x^2 + 1} \rightarrow x^2 + 1 \neq 0 \rightarrow x^2 \neq -1 \rightarrow \log \frac{1}{x} \rightarrow D_f = \mathbb{R} - \{\log \frac{1}{x}\}$$

$$ب) y = \frac{3}{x^2 - 1} \rightarrow x^2 - 1 \neq 0 \rightarrow x^2 \neq 1 \rightarrow \log \frac{1}{x} \rightarrow D_f = \mathbb{R} - \{\log \frac{1}{x}\} \cup D_f = \mathbb{R} - \{0\}$$

$$ج) y = \frac{3}{x^2 - 2} \rightarrow x^2 - 2 \neq 0 \rightarrow x^2 \neq 2 \rightarrow \log \frac{1}{x} \rightarrow D_f = \mathbb{R} - \{\log \frac{1}{x}\}$$

$$د) y = \frac{3}{x^2 - 3} \rightarrow x^2 - 3 \neq 0 \rightarrow x^2 \neq 3 \rightarrow \log \frac{1}{x} \rightarrow D_f = \mathbb{R} - \{\log \frac{1}{x}\}$$

$$الف) y = (x+1)! \quad x+1 \in W \rightarrow x \in W-1 \rightarrow x \in \frac{W-1}{x} \rightarrow D_f = \{x | x = \frac{k-1}{x}, k \in W\}$$

$$ب) y = \left(\frac{3x-2}{2x-5} \right)! \quad \frac{3x-2}{2x-5} \in W \rightarrow \frac{3x-2}{2x-5} \in \frac{W-2}{2x-5} \rightarrow \frac{3x-2}{2x-5} \in -2W+2 \rightarrow x = \frac{-2W+2}{3-2W}$$

$$D_f = \{x | x = \frac{-2W+2}{3-2W}, k \in W\}$$