سامعه شری — تکلیف ۵ — کلاس ریاضی A (جمعه‌ها)

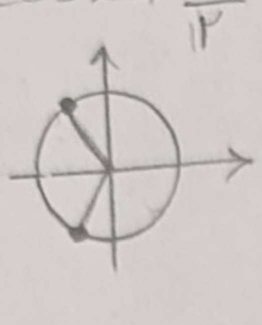
18.5

1)

الف) $\dfrac{\sin x - 2}{2\cos x + 1} \quad D_f = \mathbb{R} - \{k\pi \pm \frac{\pi}{3} \mid k \in \mathbb{Z}\}$

$2\cos x \neq -1$
$\cos x \neq -\frac{1}{2} \quad x \neq k\pi + \frac{\pi}{3}$

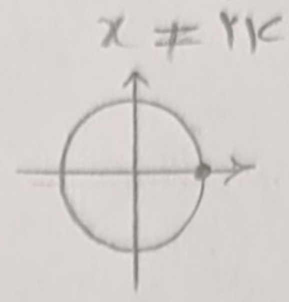
$\{2k\pi + \frac{2\pi}{3}, 2k\pi + \frac{4\pi}{3}\}$ ✓



ب) $\dfrac{\sin x + 3}{\cos x - 1} \quad D_f = \mathbb{R} - \{2k\pi \mid k \in \mathbb{Z}\}$

$\cos x - 1 \neq 0$
$\cos x \neq 1$
$x \neq 2k\pi$

1.5 ✓



2)

الف) $\dfrac{2\sin x + 1}{\tan x + 1} \quad D_f = \mathbb{R} - \{k\pi - \frac{\pi}{4} \mid k \in \mathbb{Z}\}$

$\tan x + 1 \neq 0$
$\tan x \neq -1$
$x \neq k\pi - \frac{\pi}{4}$

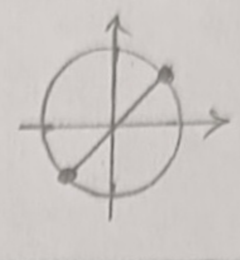
$\cos x \neq 0 \rightarrow x \neq k\pi + \frac{\pi}{2}$

1 ✓

ب) $\dfrac{\cos x + 1}{\cot x - 1} \quad D_f = \mathbb{R} - \{k\pi + \frac{\pi}{4} \mid k \in \mathbb{Z}\}$

$\cot x - 1 \neq 0$
$\cot x \neq 1$
$x \neq k\pi + \frac{\pi}{4}$

$\sin x \neq 0 \rightarrow x \neq k\pi$



3)

الف) $\sin y = x^2 - 2 \quad D_f = [-\sqrt{3}, -1] \cup [1, \sqrt{3}]$

$-1 \leq \sin y \leq 1$
$-1 \leq x^2 - 2 \leq 1$
$1 \leq x^2 \rightarrow x \leq -1$ یا $x \geq 1$
$x^2 \leq 3 \rightarrow -\sqrt{3} \leq x \leq \sqrt{3}$

2 ✓

ب) $\arccos(\sqrt{x} - 3)$

$-1 \leq \sqrt{x} - 3 \leq 1$
$-1 \leq \sqrt{x} - 3 \rightarrow \sqrt{x} \geq 2 \rightarrow x \geq 4$
$\sqrt{x} - 3 \leq 1 \rightarrow \sqrt{x} \leq 4 \rightarrow x \leq 16$

$D_f = [4, 16]$

4)

الف) $\cos y = |x| - 3 \quad D_f = [-4, -2] \cup [2, 4]$

$-1 \leq \cos y \leq 1$
$-1 \leq |x| - 3 \leq 1$
$-1 \leq |x| - 3 \rightarrow |x| \geq 2 \rightarrow x \geq 2$ یا $x \leq -2$
$|x| - 3 \leq 1 \rightarrow |x| \leq 4 \rightarrow -4 \leq x \leq 4$

2 ✓

ب) $\arcsin(x^2 + 3x + 1)$

$-1 \leq x^2 + 3x + 1 \leq 1$
$0 \leq x^2 + 3x + 2$
$(x+2)(x+1) \geq 0$
$x = -2 \quad x = -1$

$x^2 + 3x + 1 \leq 1$
$x^2 + 3x \leq 0$
$x(x+3) \leq 0$
$x = 0 \quad x = -3$

در نهایت $D_f = [-3, -2] \cup [-1, 0]$

$D_f = (-\infty, -2] \cup [-1, +\infty) \quad D_f = (-\infty, -3] \cup [0, +\infty)$

5)

الف) $y = \log_2 (x^2 - 4)$

$x^2 - 4 > 0$
$x^2 > 4 \rightarrow x > 2$ یا $x < -2$

$D_f = (-\infty, -2) \cup (2, +\infty)$

2 ✓

ب) $y = \log_2 (2 - |x|)$

$2 - |x| > 0$
$|x| < 2 \rightarrow -2 < x < 2$

$D_f = (-2, 2)$

6)

الف) $y = \log_{x-3} (5 - x)$

$5 - x > 0 \rightarrow x < 5$
$x - 3 > 0 \rightarrow x > 3$
$x - 3 \neq 1 \rightarrow x \neq 4$

$D_f = (3, 5) - \{4\} \rightarrow = (3, 4) \cup (4, 5)$

2 ✓

ب) $\log_{x+3} (x^2 - 1)$

I $\quad x^2 - 1 > 0 \rightarrow x^2 > 1 \rightarrow x > 1, x < -1$
II $\quad x + 3 > 0 \quad x > -3 \quad x + 3 \neq 1 \quad x \neq -2$

$-3 < x < -1 \} (-3, -2) \cup (-2, -1)$
$x \neq -2$

$D_f = (-3, -2) \cup (-2, -1) \cup (1, +\infty)$

7)

الف) $\log \dfrac{x^2 - 4x + 3}{x - 3}$

$\dfrac{x^2 - 4x + 3}{x - 3} > 0 \rightarrow \dfrac{(x-1)(x-3)}{x-3} > 0$

$x \neq 3$

$x - 1 > 0 \rightarrow x > 1$

$D_f = (1, +\infty) - \{3\}$

2 ✓

ب) $\log_{x+5} \dfrac{x+3}{x-2}$

$\dfrac{x+3}{x-2} > 0$
$\hookrightarrow x \neq 2$

$x + 5 > 0 \quad x + 5 \neq 1$
$x > -5 \quad x \neq -4$
$x \in (-5, +\infty)$

$x + 3 > 0, \; x - 2 > 0 \rightarrow x > 2$
$x + 3 < 0, \; x - 2 < 0 \rightarrow x < -3$
$x \in (-\infty, -3) \cup (2, +\infty)$

$(-\infty, -3) \cap (-5, \infty) = (-5, -3)$
$-\{-4\} \rightarrow (-5, -4) \cup (-4, -3)$
$(2, \infty) \cap (-5, \infty) = (2, \infty)$

جواب $D_f = (-5, -4) \cup (-4, -3) \cup (2, +\infty)$

۸) ب) $\log(2\log_{2^2}^{x} - 1)$

$2\log_{2^2}^{x} - 1 > 0 \rightarrow 2\log_{2^2}^{x} > 1 \rightarrow \log_{2^2}^{x} > \frac{1}{2}$

$x > \sqrt{2^0}$ $\quad D_f = (\sqrt{2^0}, \infty)$

الف) $y = \sqrt{2^0 - \log_2^{(x-2)}}$ $\quad x-2 > 0 \rightarrow x > 2$ I

$2^0 - \log_2^{(x-2)} \geq 0$

$\log_2^{(x-2)} \leq 2^0$

$(x-2) \leq 1^0$

$x \leq 10$ II

$D_1 = (2, +\infty)$

$D_{II} = (-\infty, 10]$

$D_f = I \cap II = (2, 10]$

۹) ب) $y = \frac{2}{4^x - 1}$ $\quad 4^x - 1 \neq 0$

$4^x \neq 1$

$x \neq 0$

$D_f = \mathbb{R} - \{0\}$

الف) $y = \frac{3}{4^x + 1}$ $\quad 4^x + 1 \neq 0$

$4^x \neq -1 \rightarrow$ امکان پذیرست

$4^x > 0 \rightarrow 4^x + 1 > 0$

خروجی نمی‌شود :

$D_f = \mathbb{R}$

۱۰) ب) $y = \frac{2}{4^x - 4}$ $\quad 4^x - 4 \neq 0$

$4^x \neq 4$

$x \neq \log_4^4$

$D_f = \mathbb{R} - \{\log_4^4\}$

ج) $y = \frac{3}{4^x - 2}$ $\quad 4^x - 2 \neq 0$

$4^x \neq 2$

$x \neq \frac{1}{2}$

$D_f = \mathbb{R} - \{\frac{1}{2}\}$

۱۱) الف) $y = (4x+1)!$

$4x + 1 \in W$

$4x \in W - 1$

$x \in \frac{W-1}{4}$

$\{x | x = \frac{k-1}{4}, k \in W\}$

۱۲) ب) $(\frac{4x-2}{2x-5})!$

$\frac{4x-2}{2x-5} \in W$

$4x - 2 \in 2xW - 5W$

$4x - 2xW \in -5W - 2$

$x(4 - 2W) \in -5W - 2$

$x \in \frac{-5W-2}{4-2W} \rightarrow \frac{(-1)(5W+2)}{(-1)(2W-4)}$

$\{x | x = \frac{5k+2}{2k-4}, k \in W\}$