

الف $y = \frac{\sin x - 1}{1 + \cos x}$

$1 + \cos x \neq 0 \rightarrow \cos x \neq -1 \rightarrow \cos x \neq -\frac{1}{1}$
 $Df = \mathbb{R} - \{k\pi + \frac{\pi}{2}\}$



①

ب $y = \frac{\sin x + 1}{\cos x - 1}$

$\cos x - 1 \neq 0 \rightarrow \cos x \neq 1$
 $Df = \mathbb{R} - \{k\pi\}$



الف $y = \frac{\sin x + 1}{\cos x}$

$\tan x + 1 \neq 0 \rightarrow \tan x \neq -1$
 $\cos x \neq 0 \rightarrow Df = \mathbb{R} - \{k\pi, k\pi - \frac{\pi}{2}\}$



②

ب $y = \frac{\cos x + 1}{\sin x}$

$\cot x - 1 \neq 0 \rightarrow \cot x \neq 1$
 $\sin x \neq 0 \rightarrow Df = \mathbb{R} - \{k\pi, k\pi + \frac{\pi}{2}\}$



الف $\sin y = x^2 - 2$

$-1 \leq x^2 - 2 \leq 1 \rightarrow 1 \leq x^2 \leq 3 \rightarrow \sqrt{1} \leq x \leq \sqrt{3} \text{ or } -\sqrt{3} \leq x \leq -1$
 $Df = [-\sqrt{3}, -1] \cup [1, \sqrt{3}]$

ب $\arccos(\sqrt{x} - 2)$

$-1 \leq \sqrt{x} - 2 \leq 1 \rightarrow 1 \leq \sqrt{x} \leq 3 \rightarrow 1 \leq x \leq 9$
 $Df = [1, 9]$

الف $\cos y = |x| - 2$

$-1 \leq |x| - 2 \leq 1 \rightarrow 1 \leq |x| \leq 3 \rightarrow \begin{cases} 1 \leq x \leq 3 \\ -3 \leq x \leq -1 \end{cases} \rightarrow Df = [-3, -1] \cup [1, 3]$

ب $\arcsin(x^2 + 3x + 1)$

$-1 \leq x^2 + 3x + 1 \leq 1 \rightarrow \begin{cases} x^2 + 3x + 1 \leq 1 \rightarrow x^2 + 3x \leq 0 \text{ (I)} \rightarrow x(x+3) \leq 0 \\ x^2 + 3x + 1 \geq -1 \rightarrow x^2 + 3x + 2 \geq 0 \text{ (II)} \rightarrow (x+1)(x+2) \geq 0 \end{cases}$

$I = [-3, 0]$

$II = (-\infty, -2] \cup [-1, +\infty)$

$\frac{-3}{+} \frac{0}{-} \frac{+}{+} \text{ (I)}$

$\frac{-2}{+} \frac{-1}{-} \frac{+}{+} \text{ (II)}$

$Df = (I) \cap (II) \rightarrow Df = [-3, -2] \cup [-1, 0]$

الف $y = \log_r x^{x-2}$

$x^{x-2} > 0 \rightarrow x^{x-2} > 1 \rightarrow x > 2$
 $Df = (-\infty, -2) \cup (2, +\infty)$

ب $\log_r r - |x|$

$r - |x| > 0 \rightarrow 2 > |x| \rightarrow -2 < x < 2$
 $Df = (-2, 2)$

الف $\log_{x-2} 0-x$

$0-x > 0 \rightarrow x < 0 \rightarrow (-\infty, 0)$
 $x-2 > 0 \rightarrow x > 2 \rightarrow (2, +\infty)$
 $x-2 \neq 1 \rightarrow x \neq 3 \rightarrow Df = (-\infty, 0) \cup (2, +\infty) - \{3\}$

ب $\log_{x+2} x^2-1$

$x^2-1 > 0 \rightarrow x^2 > 1 \rightarrow x > 1 \text{ or } x < -1$
 $x+2 > 0 \rightarrow x > -2 \rightarrow (-2, +\infty)$
 $x+2 \neq 1 \rightarrow x \neq -1$
 $Df = (-\infty, -1) \cup (1, +\infty)$

$Df = \bigcap \rightarrow (-2, -1) \cup (1, +\infty) - \{-1\}$

الف) $y = \log \frac{x^r - rx + r}{x-r} \rightarrow \frac{x^r - rx + r}{x-r} > 0 \rightarrow \frac{(x-r)(x-1)}{x-r}$

$\frac{x}{r}$
 $\begin{array}{c|c|c} - & 0 & + \\ \hline 1 & r & \end{array}$
 $Df = (1, +\infty)$

ب) $y = \log \frac{x+r}{x+a}$

$\frac{x+r}{x-r} > 0$
 $\begin{array}{c|c|c} -r & r & \\ \hline + & - & 0 & + \end{array}$

$x+a > 0 \rightarrow x > -a$

$x+a \neq 1 \rightarrow x \neq -r$

$(-\infty, +\infty) - \{-r\}$

$((-\infty, -r) \cup (r, +\infty))$

$x-r \neq 0$

$x \neq r$

$\rightarrow Df = (-\infty, -r) \cup (r, +\infty) - \{-r\}$

الف) $y = \sqrt{r \cdot \log_r(x-r)}$

$r - \log_r(x-r) > 0 \rightarrow r > \log_r(x-r)$

$1 > x-r$

$x-r > 0 \rightarrow x > r$

$Df = (r, 1]$

$1 > x$

ب) $y = \log(r \log_r^x - 1) \quad x > 0$

$r \log_r^x - 1 > 0 \rightarrow r \log_r^x > 1 \rightarrow \log_r^x > \frac{1}{r} \rightarrow x > r^{\frac{1}{r}} \rightarrow x > \sqrt[r]{r} \quad Df = (\sqrt[r]{r}, +\infty)$

الف) $y = \frac{r}{r^x + 1}$

$r^x + 1 \neq 0 \rightarrow r^x \neq -1 \quad Df = \mathbb{R}$

ب) $y = \frac{r}{\frac{x}{r-1}}$

$r^x - 1 \neq 0 \rightarrow r^x \neq 1 \rightarrow x \neq 0 \rightarrow Df = \mathbb{R} - \{0\}$

ج) $y = \frac{r}{r^{\frac{x}{r}} - r}$

$r^{\frac{x}{r}} - r \neq 0 \rightarrow r^{\frac{x}{r}} \neq r \rightarrow x \neq \log_r r \rightarrow Df = \mathbb{R} - \{\log_r r\}$

د) $y = \frac{r}{\frac{x}{r} - r}$

$r^{\frac{x}{r}} - r \neq 0 \rightarrow r^{\frac{x}{r}} \neq r \rightarrow x \neq \log_r r \rightarrow Df = \mathbb{R} - \{\log_r r\}$

الف) $y = (rx+1)! \rightarrow rx+1 \in \omega \rightarrow rx \in \omega-1 \rightarrow x \in \frac{\omega-1}{r}$

$Df = \{x \mid x = \frac{k-1}{r}, k \in \omega\}$

ب) $y = \left(\frac{rx-r}{rx-a}\right)!$

$\frac{rx-r}{rx-a} \in \omega$

$\rightarrow rx-r = r\omega x - a\omega \rightarrow rx - r\omega x = r - a\omega$

$x(r-r\omega) = r - a\omega$

$x = \frac{r-a\omega}{r-r\omega}$

$Df = \{x \mid x = \frac{r-a\omega}{r-r\omega}, k \in \omega\}$