

د: $\sin^{-1}$

19.8

A: $\sin^{-1}$ (0, 1), $\cos^{-1}$ (1, 0)

الف) $y = \frac{\sin n - 2}{\cos n + 1}$ $\rightarrow \begin{cases} \cos n + 1 \neq 0 \\ \cos n \neq -\frac{1}{1} \end{cases}$
 $\hookrightarrow n \neq \frac{\pi}{2}, \frac{3\pi}{2}$

$D_f = \mathbb{R} - \left\{ \frac{\pi}{2} + k\pi, \frac{3\pi}{2} + k\pi \right\}$



ب) $y = \frac{\sin n + 2}{\cos n - 1}$

$\hookrightarrow \cos n - 1 \neq 0 \rightarrow \cos n \neq 1$
 $n \neq 0, 2\pi$



الف) $y = \frac{\cos n + 1}{\tan n + 1}$ $\rightarrow \begin{cases} \tan n + 1 \neq 0 \\ \tan n \neq -1 \end{cases}$
 $\hookrightarrow n \neq \frac{3\pi}{4}, \frac{7\pi}{4}$

(1) $\tan n + 1 \neq 0 \Rightarrow \tan n \neq -1$
 $n \neq \frac{3\pi}{4}, \frac{7\pi}{4}$

$D_f = \mathbb{R} - \left\{ \frac{3\pi}{4} + k\pi, \frac{7\pi}{4} + k\pi \right\}$



ب) $y = \frac{\cos n + 1}{\cot n - 1}$ $\rightarrow \begin{cases} \cot n - 1 \neq 0 \\ \cot n \neq 1 \end{cases}$
 $\hookrightarrow n \neq \frac{\pi}{4}, \frac{5\pi}{4}$

(2) $\cot n - 1 \neq 0 \Rightarrow \cot n \neq 1 \Rightarrow n \neq \frac{\pi}{4}, \frac{5\pi}{4}$

$D_f = \mathbb{R} - \left\{ \frac{\pi}{4}, \frac{5\pi}{4} \right\}$



الف) $\sin y = x^2 - 2 \Rightarrow -1 \leq x^2 - 2 \leq 1$ $\rightarrow \begin{cases} x^2 - 2 \leq 1 \\ x^2 - 2 \geq -1 \end{cases}$

$\sin y \in [-1, 1]$

$\hookrightarrow \begin{cases} x^2 \leq 3 \\ x^2 \geq 1 \end{cases} \Rightarrow \begin{cases} -\sqrt{3} \leq x \leq \sqrt{3} \\ x \geq 1 \text{ or } x \leq -1 \end{cases}$

$(1) \cap (2) \Rightarrow D_f = [-\sqrt{3}, -1] \cup [1, \sqrt{3}]$

ب) $y = \arccos(\sqrt{n} - 2)$

$\Rightarrow -1 \leq \sqrt{n} - 2 \leq 1$
 $\hookrightarrow \begin{cases} \sqrt{n} - 2 \leq 1 \\ \sqrt{n} - 2 \geq -1 \end{cases} \Rightarrow \begin{cases} \sqrt{n} \leq 3 \\ \sqrt{n} \geq 1 \end{cases}$
 $n \leq 9$
 $n \geq 1$

$(1) \cap (2) \Rightarrow D_f = [1, 9]$

الف) $\cos y = |x| - 2 \Rightarrow -1 \leq |x| - 2 \leq 1 \Rightarrow \begin{cases} |x| - 2 \leq 1 \\ |x| - 2 \geq -1 \end{cases}$

$x^2 + 2x + 1 \leq 0$
 $x(x+2) \leq 0$
 $\hookrightarrow [-2, 0]$

(2) $|x| \geq 1$
 $x \geq 1$ or $x \leq -1$

$(1) \cap (2) \Rightarrow D_f = [-2, -1] \cup [1, 2]$

ب) $y = \arcsin(x^2 + 2x + 1)$

$\hookrightarrow -1 \leq x^2 + 2x + 1 \leq 1$
 $x^2 + 2x + 2 \geq 0$
 $(x+1)^2 \geq -1 \Rightarrow (-\infty, -1] \cup [-1, \infty)$

$(1) \cap (2) \Rightarrow D_f = [-2, -1] \cup [-1, 0]$

الف) $y = \log_r n^r - r \Rightarrow n^r - r > 0$
 $n^r > r \Rightarrow n > r$ } $Df = (-\infty, r) \cup (r, +\infty)$ (3)

ب) $y = \log_r r - \ln |$
 $\Rightarrow r - \ln | > 0$
 $|\ln | < r \Rightarrow -r < n < r$ } $Df = (-r, r)$ (4) ✓

الف) $y = \log_{n-r} \frac{a-n}{n-r} \Rightarrow \textcircled{1} \rightarrow a-n > 0$
 $n < a$
 $\textcircled{2} \rightarrow n-r > 0 \Rightarrow n > r$ } $Df = (r, a) - \{r\} \Rightarrow \textcircled{1} \cap \textcircled{2}$ (5)

ب) $y = \log_{n+r} \frac{n^r-1}{n+r}$
 $\textcircled{1} \Rightarrow n^r-1 > 0 \Rightarrow n^r > 1 \Rightarrow n > 1$
 $\textcircled{2} \Rightarrow n+r > 0 \Rightarrow n > -r$
 $n+r \neq 1 \Rightarrow n \neq -r$ } $Df = (-r, -1) \cup (1, +\infty) - \{-r\}$ (6) ✓

الف) $y = \log \frac{x^r - rx + r}{n-r} : \frac{(n-r)(n-1)}{n-r} > 0 \Rightarrow \frac{1}{\frac{1}{n} + \frac{1}{r} + 1}$ } $Df = (1, r) \cup (r, +\infty)$ (7)

ب) $y = \log \frac{n+r}{n-r} \Rightarrow \textcircled{1} \rightarrow \frac{n+r}{n-r} > 0 \Rightarrow \frac{-r}{\frac{1}{n} + \frac{1}{r} + 1} \Rightarrow \textcircled{1} \rightarrow (-\infty, -r) \cup (r, +\infty)$
 $\textcircled{2} \rightarrow n+r > 0 \Rightarrow n > -r$ } $Df = (-\infty, -r) \cup (r, +\infty)$ (8) ✓

الف) $y = \sqrt{r} - \log_r (x-r) \Rightarrow \textcircled{1} \rightarrow x-r > 0 \Rightarrow x > r$ } $Df = (r, +\infty)$ (9)

ب) $y = \log(r \log_r^n - 1) \Rightarrow \textcircled{1} \rightarrow r \log_r^n > 1 \Rightarrow \log_r^n > \frac{1}{r} \Rightarrow n > \sqrt[r]{r}$
 $\textcircled{2} \rightarrow r \log_r^n - 1 > 0 \Rightarrow r \log_r^n > 1 \Rightarrow \log_r^n > \frac{1}{r} \Rightarrow n > \sqrt[r]{r}$ } $Df = (\sqrt[r]{r}, +\infty)$ (10) ✓

الف) $y = \frac{r}{r^n + 1} \Rightarrow \frac{r^n}{r^n + 1} \neq 0$
 $\frac{r^n}{r^n + 1} > 0 \Rightarrow r^n > 0$ } $Df = \mathbb{R}$ (11)

ب) $y = \frac{r}{r^n - 1} \Rightarrow \frac{r^n}{r^n - 1} \neq 0$
 $\frac{r^n}{r^n - 1} > 0 \Rightarrow r^n > 1 \Rightarrow n > 0$ } $Df = \mathbb{R} - \{0\}$ (12)

ج) $y = \frac{r}{r^n - r} \Rightarrow \frac{r^n - r}{r^n - r} \neq 0$
 $\frac{r^n - r}{r^n - r} \neq 0 \Rightarrow n \neq \log_r r = 1$ } $Df = \mathbb{R} - \{1\}$ (13) ✓

الف) $y = (f_{n+1})!$
 $f_{n+1} \in \mathbb{W}$
 $f_n \in \mathbb{W} - 1$
 $n \in \frac{\mathbb{W} - 1}{r}$ } $Df = \{n | n \in \frac{\mathbb{W} - 1}{r}, K \in \mathbb{W}\}$ (14) ✓

ب) $y = \left(\frac{r^n - r}{r^n - a}\right)!$
 $\frac{r^n - r}{r^n - a} \in \mathbb{W} \Rightarrow n \in \frac{r - a\mathbb{W}}{r - r\mathbb{W}}$
 $r^n - r \in r\mathbb{W} - a\mathbb{W}$
 $r^n - r\mathbb{W} \in r - a\mathbb{W}$ } $Df = \{n | n \in \frac{r - a\mathbb{W}}{r - r\mathbb{W}}, K \in \mathbb{W}\}$ (15)