

الف) $y = \frac{\sin x - 1}{\cos x + 1}$ $\int \frac{1}{\cos x + 1} \neq 0 \Rightarrow \cos x \neq -1 \Rightarrow \cos x \neq -\frac{1}{1} \Rightarrow D_f = \mathbb{R} - \left\{ k\pi + \frac{\pi}{2}, k\pi + \frac{3\pi}{2} \right\}$

ب) $y = \frac{\sinh x + 1}{\cosh x - 1}$ $\int \frac{1}{\cosh x - 1} \neq 0 \Rightarrow \cosh x \neq 1 \Rightarrow D_f = \mathbb{R} - \{ k\pi \}$

الف) $y = \frac{1 \sin x + 1}{\tan x + 1}$ $\int \frac{1}{\tan x + 1} \neq 0 \Rightarrow \tan x \neq -1 \Rightarrow D_f = \mathbb{R} - \left\{ k\pi + \frac{\pi}{4}, k\pi + \frac{5\pi}{4} \right\}$

ب) $y = \frac{\cos x + 1}{\cot x - 1}$ $\int \frac{1}{\cot x - 1} \neq 0 \Rightarrow \cot x \neq 1 \Rightarrow D_f = \mathbb{R} - \left\{ k\pi + \frac{\pi}{4}, k\pi \right\}$

الف) $\sin y = x^1 - 1 \Rightarrow 1 \geq x^1 - 1 \geq -1 \Rightarrow 1^2 \geq x^1 \geq -1 \Rightarrow -\sqrt{1} \leq x \leq -1 \cup 1 \geq x \geq 1$
 $D_f = [-1, 1] \cup [1, 1]$

ب) $y = \arccos(\sqrt{x} - 1) \Rightarrow 1 \geq \sqrt{x} - 1 \geq -1 \Rightarrow \sqrt{x} \geq 0 \Rightarrow x \geq 0 \Rightarrow 1 \geq \sqrt{x} \Rightarrow x \leq 1$
 $D_f = [0, 1]$

الف) $\cos y = |x| - 1 \Rightarrow 1 \geq |x| - 1 \geq -1 \Rightarrow 1^2 \geq |x| \geq 1^2 \Rightarrow x \geq 1 \cup -1 \geq x \geq -1$
 $D_f = [1, 1] \cup [-1, -1]$

ب) $y = \arcsinh(x^1 + 1) \Rightarrow 1 \geq x^1 + 1 \geq -1 \Rightarrow 0 \geq x \geq -2$
 $\Rightarrow D_f = [-2, 0]$

الف) $y = \log_{10} x^1 - 1 \Rightarrow x^1 - 1^2 \geq 0 \Rightarrow x^1 \geq 1^2 \Rightarrow x \geq 1 \Rightarrow D_f = (1, +\infty) \cup (-\infty, -1)$
 ب) $y = \log_{10} x^1 - 1 \Rightarrow x^1 - 1^2 \geq 0 \Rightarrow |x| < 1 \Rightarrow -1 < x < 1 \Rightarrow D_f = (-1, 1)$

الف) $y = \log_{10} x^1 - x \Rightarrow 0 - x \geq 0 \Rightarrow x < 0$
 $x - 1^2 \geq 0 \Rightarrow x \geq 1^2, x - 1^2 \neq 1 \Rightarrow x \neq 1^2 \Rightarrow D_f = (1, \infty) - \{1^2\}$

ب) $y = \log_{10} x^1 - 1 \Rightarrow x^1 \geq 1 \Rightarrow x < -1 \cup x > 1 \Rightarrow D_f = (-1, -1) \cup (1, +\infty) - \{-1\}$
 $x + 1^2 \geq 0 \Rightarrow x \geq -1^2, x + 1^2 \neq 1 \Rightarrow x \neq -1^2 \Rightarrow D_f = (-1, -1) \cup (1, +\infty) - \{-1\}$

✓

$$\text{الف) } y = \log \frac{x^2 - 4x + 3}{x - 3} = \log \frac{(x-3)(x-1)}{x-3} = \log (x-1) \Rightarrow x > 1 \quad \text{✓}$$

$$D_f = (1, +\infty) - \{3\}$$

$$\text{ب) } y = \log \frac{x+3}{x-1} \rightarrow \frac{x+3}{x-1} > 0 \Rightarrow x < -3 \text{ or } x > 1 \quad \text{✓}$$

$$x+3 > 0 \Rightarrow x > -3, x-1 \neq 0, x \neq 1 \Rightarrow D_f = (-3, -1) \cup (1, +\infty) - \{1\}$$

$$\text{الف) } y = \sqrt{2 - \log_p(x-1)} \rightarrow x-1 > 0 \Rightarrow x > 1 \quad (I)$$

$$D_f = (I) \cap (II) = (1, 10]$$

$$\text{ب) } y = \log(2 \log_p x - 1) \rightarrow x > 0 \quad (I) \quad \text{✓}$$

$$2 \log_p x - 1 > 0 \Rightarrow \log_p x > \frac{1}{2} \Rightarrow x > p^{\frac{1}{2}} \Rightarrow x > \sqrt{p} \quad (II)$$

$$D_f = (I) \cup (II) = (\sqrt{p}, +\infty)$$

$$\text{الف) } y = \frac{p}{p^x + 1} \rightarrow p^x + 1 \neq 0 \Rightarrow p^x \neq -1 \Rightarrow D_f = \mathbb{R}$$

$$\text{ب) } y = \frac{p}{p^x - 1} \rightarrow p^x - 1 \neq 0 \Rightarrow p^x \neq 1 \Rightarrow x \neq 0 \Rightarrow D_f = \mathbb{R} - \{0\}$$

$$\text{ج) } y = \frac{p}{p^x - p} \rightarrow p^x - p \neq 0 \Rightarrow p^x \neq p \Rightarrow x \neq \log_p p \Rightarrow D_f = \mathbb{R} - \{\log_p p\}$$

$$\text{د) } y = \frac{p}{p^x - p^3} \rightarrow p^x - p^3 \neq 0 \Rightarrow p^x \neq p^3 \Rightarrow x \neq \log_p p^3 \Rightarrow D_f = \mathbb{R} - \{\log_p p^3\}$$

$$\text{الف) } y = (kx+1)! \Rightarrow kx+1 \in \mathbb{N} \Rightarrow kx \in \mathbb{N} - 1 \Rightarrow x \in \frac{\mathbb{N}-1}{k} \Rightarrow D_f = \{x / x \in \frac{k-1}{k}, k \in \mathbb{N}\}$$

$$\text{ب) } y = \left(\frac{kx-1}{1-x} \right)! \Rightarrow \frac{kx-1}{1-x} \in \mathbb{N} \Rightarrow kx-1 \in 1-x\mathbb{N} \Rightarrow kx-1-x\mathbb{N} \in 1-\mathbb{N}$$

$$\Rightarrow x \in \frac{1-\mathbb{N}}{k-1} \Rightarrow D_f = \{x / x \in \frac{1-k}{k-1}, k \in \mathbb{N}\}$$