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الف) $y = \sqrt{4 - \sqrt{2-x}} \rightarrow$

$\begin{cases} 2-x \geq 0 \rightarrow x \leq 2 \\ 4 - \sqrt{2-x} \geq 0 \rightarrow 4 \geq \sqrt{2-x} \rightarrow 16 \geq 2-x \rightarrow 14 \geq -x \rightarrow -14 \leq x \end{cases}$

$D_f = [-14, 2]$

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ب) $y = \sqrt{3 - \sqrt{x-2}} \rightarrow$

$\begin{cases} x-2 \geq 0 \rightarrow x \geq 2 \\ 3 - \sqrt{x-2} \geq 0 \rightarrow 3 \geq \sqrt{x-2} \rightarrow 9 \geq x-2 \rightarrow 11 \geq x \end{cases}$

$D_f = [2, 11]$

الف) $y = \sqrt{4-2x^2} \rightarrow$

$4-2x^2 \geq 0 \rightarrow 2(2-x^2)$

$\rightarrow 2 \geq x^2 \rightarrow \pm \sqrt{2} \geq x \rightarrow$

$D_f = [-\sqrt{2}, +\sqrt{2}]$

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ب) $y = \sqrt{3(|x|-9)} \rightarrow$

$3(|x|-9) \geq 0 \rightarrow 3|x| \geq 9 \rightarrow |x| \geq 3 \rightarrow \begin{cases} x \geq 3 \\ x \leq -3 \end{cases} D_f = (-\infty, -3] \cup [3, +\infty)$

الف) $y_1 = \sqrt{\frac{|x|+1}{|x|-3}}$

$|x|-3 \neq 0 \rightarrow |x| \neq 3 \rightarrow x \neq \pm 3 \rightarrow D_f = \mathbb{R} - \{\pm 3\}$

ب) $y = \sqrt{\frac{\sqrt{x}+1}{\sqrt{x}-3}}$

$\sqrt{x} \neq 3 \rightarrow x \neq 9 \quad D_f = [0, +\infty) - \{9\}$

الف) $y = \frac{\sqrt{4-|x|}}{|x|+2}$

$4-|x| \geq 0 \rightarrow 4 \geq |x| \rightarrow |x| \leq 4 \rightarrow -4 \leq x \leq 4$

$D_f = [-4, 4]$

ب) $y = \frac{\sqrt{4-x^2}}{|x|-1}$

$4-x^2 \geq 0 \rightarrow x^2 \leq 4 \rightarrow -2 \leq x \leq 2$
 $|x|-1 \neq 0 \rightarrow |x| \neq 1 \rightarrow x \neq \pm 1$
 $D_f = [-2, 2] - \{-1, 1\}$

الف) $y = \frac{x+1}{\sqrt{x+|x|}}$

$x+|x| > 0 \rightarrow |x| > -x \rightarrow D_f = \mathbb{R}^+ \subseteq (0, +\infty)$

ب) $y = \frac{1}{\sqrt{x|x|}}$

$x|x| > 0 \rightarrow D_f = \mathbb{R}^+ \subseteq (0, +\infty)$

الف) $y = \sqrt{2-x} \rightarrow 2-x \geq 0 \rightarrow x \leq 2 \rightarrow x < 2 \rightarrow D_f = (-\infty, 2)$

ب) $y = \frac{1}{\sqrt{2-x}} \rightarrow 2-x > 0 \rightarrow x < 2 \rightarrow x < 2 \rightarrow D_f = (-\infty, 2)$

الف) $y = \frac{1}{x[x]}$ $\left\{ \begin{array}{l} x[x] \neq 0 \\ x \neq 0 \\ [x] \neq 0 \end{array} \right\} \rightarrow D_f = \mathbb{R} - [0, 1)$

ب) $y = \frac{1}{\sqrt{-x[x]}}$ $-x[x] > 0 \rightarrow x[x] < 0 \rightarrow D_f = \emptyset$ *لا يوجد*

الف) $y = \sqrt{\left[x - \frac{1}{x}\right] + \left[x + \frac{1}{x}\right]} \rightarrow \left[x - \frac{1}{x}\right] + \left[x + \frac{1}{x}\right] + 1 \geq 1 \rightarrow \left[x + \frac{1}{x}\right] \geq 1 \rightarrow \left[x + \frac{1}{x}\right] \geq \frac{1}{x}$ $D_f = \left[\frac{1}{x}, +\infty\right) \leftarrow x \geq \frac{1}{x} \leftarrow x + \frac{1}{x} \geq 1$ (A)

ب) $y = \sqrt{\left[x - \frac{1}{x}\right] + \left[-x + \frac{1}{x}\right]} \rightarrow \left\{ \begin{array}{l} a \in \mathbb{Z} \rightarrow [a] + [-a] = 0 \\ a \notin \mathbb{Z} \rightarrow [a] + [-a] = -1 \end{array} \right. \rightarrow x - \frac{1}{x} \in \mathbb{Z} \quad D_f = \left\{ x \mid x = k + \frac{1}{x}, k \in \mathbb{Z} \right\}$

الف) $y = \frac{1}{x \sin^2 x - 1} \rightarrow x \sin^2 x - 1 \neq 0 \rightarrow \sin^2 x \neq \frac{1}{x} \rightarrow \sin x \neq \pm \sqrt{\frac{1}{x}} \quad \mathbb{R} - \left\{ k\pi \pm \frac{\pi}{2} \right\}$

ب) $y = \frac{\cot x + 1}{\tan x + 1} \rightarrow \frac{\frac{\cos x}{\sin x} + 1}{\frac{\sin x}{\cos x} + 1} \Rightarrow \frac{\cos x + \sin x}{\sin x + \cos x} \Rightarrow \frac{\cos x}{\sin x} \neq -1 \Rightarrow \tan x \neq -1$ $D_f = \mathbb{R} - \left\{ k\pi, k\pi + \frac{\pi}{2}, k\pi - \frac{\pi}{2} \right\}$

الف) $y = \sqrt{2 \sin x - 1} = \sqrt{2 \sin x} \cdot 1 \geq 0 \rightarrow \sin x \geq \frac{1}{2}$ $D_f = \left[2k\pi + \frac{\pi}{6}, 2k\pi + \frac{5\pi}{6} \right]$

ب) $y = \sqrt{1 - 2 \cos x} = \sqrt{2 \cos x} \cdot 1 \leq 0 \rightarrow \cos x \leq \frac{1}{2} \rightarrow D_f = \left[2k\pi + \frac{\pi}{3}, 2k\pi + \frac{5\pi}{3} \right]$