دهم ریاضی A — تکلیف شماره ۲۸ — فاطمه اجری

۱- $f(x)=\begin{cases} x^2+2x & ; x\geq a \\ ax-4 & ; x\leq a \end{cases}$ $\quad a=?$

$x=a \rightsquigarrow a^2+2a=a^2-4 \to 2a=-4 \to a=-2$

۲- $f(x)=\dfrac{x^2+a}{2x-b}$, $g(x)=2x+b \rightsquigarrow (2,3) \Rightarrow f(1)=?$

$f(2)=\dfrac{4+a}{4-b}=3 \rightsquigarrow 4+a=12-(3b) \rightsquigarrow 4+a=15 \to a=11$

$g(2)=4+b=3 \rightsquigarrow b=-1 \Rightarrow f(x)=\dfrac{x^2+11}{2x+1}$

$f(1)=\dfrac{12}{3}=4$

۳- $f(x)=\dfrac{4x+1}{2x^2+ax+b}$ $\quad D=\mathbb{R}-\{-1,4\} \rightsquigarrow f(1)=?$

↳ $2x^2+ax+b\neq 0 \Rightarrow$

$x=-1 \to (2-a+b=0)\times 4$; $8-4a+4b=0$

$x=4 \to 32+4a+b=0$

$5b=-40$ ↳ $b=-8$, $a=-6$

$f(x)=\dfrac{4x+1}{2x^2-6x-8} \rightsquigarrow f(1)=-\dfrac{5}{12}$

۴- $f(x)=\dfrac{x^3-\sqrt{3}}{-4x^2+ax+b}$ $\quad D=\mathbb{R}-\{-1\}$ $\quad a+b=?$

$-4x^2+ax+b\neq 0$ ↳ ریشه مضاعف $\Rightarrow -4(x+1)^2$ [مربع کامل]

$-4(x^2+2x+1) = -4x^2-8x-4$ ($a=-8$, $b=-4$) $\Leftarrow$ $a+b=-8-4=-12$

۵- $f(x)=\dfrac{2x}{(x-1)(x^2+mx+1)}$ $\quad D=\mathbb{R}-\{1\}$ (بی‌ریشه یا ریشه مضاعف) $\quad$ حدود $m=?$

(ریشه)

↳ x^2+mx+1 (بی‌ریشه) $\to \Delta=m^2-4<0 \to m^2<4 \to -2<m<2$ ①

$x^2+mx+1 \overset{x=1}{\rightsquigarrow} 1+m+1=0 \to m=-2$ ②

اجتماع ①,② $\longrightarrow$ $-2\leq m<2$

$f(x)=\sqrt{4-\frac{1}{x^2}}$ $\rightarrow x\neq 0$, $D_f=?$ -۶

$4-\frac{1}{x^2}\geq 0 \rightarrow \left(4\geq \frac{1}{x^2}\right)\times x^2 \rightarrow 4x^2\geq 1 \rightarrow x^2\geq \frac{1}{4}$ ✓ $x\geq \frac{1}{2}$, $x\leq -\frac{1}{2}$

$D=\left(-\infty,\frac{-1}{2}\right]\cup\left[\frac{1}{2},+\infty\right)$

$f(x)=\sqrt{mx^2+2mx+1}$ $D=\mathbb{R}$ $m=?$ -۷

$mx^2+2mx+1=0$

↳ $m=0 \rightsquigarrow 1\geq 0$ ✓ | $m\neq 0 \rightsquigarrow \Delta\leq 0 \Rightarrow 4m^2-4m\leq 0 \xrightarrow{\div 4} m(m-1)\leq 0$

$\begin{array}{ccccc} & 0 & & 1 & \\ + & \| & - & \| & + \end{array}$ $\rightsquigarrow [0,1]\in m$

$f(x)=\begin{cases}\frac{4x^2-1}{2x-1} & ; x\neq a\\ 4x+k & ; x=\frac{1}{2}\end{cases}$ $g(x)=2x+1$ $a+k=?$ -۸

$g\left(\frac{1}{2}\right)=2$ / $f\left(\frac{1}{2}\right)=2+k \rightsquigarrow 2+k=2 \rightarrow k=0$

$\frac{4\left(\frac{1}{2}\right)-1}{1-1}\rightarrow \frac{0}{0} \rightsquigarrow x\neq\frac{1}{2} \Rightarrow a=\frac{1}{2}$ $a+k=\frac{1}{2}$

$f(x)=\begin{cases}\frac{9x^2-4}{3x+2} & ; x\neq -\frac{2}{3}\\ 2ax+2 & ; x=\frac{-2}{3}\end{cases}$ $g(x)=3x+b$; $a-b=?$ -۹

$x=1 \rightarrow f(1)=\frac{5}{5}=1$, $g(1)=2+b \Rightarrow 2+b=1 \rightarrow b=-2$

$x=\frac{-2}{3} \rightarrow f\left(\frac{-2}{3}\right)=-2a+2=-4 \rightarrow +2a=+6 \rightarrow a=3$

$a-b=3-(-2)=5$

$f(x)=\begin{cases}\frac{x^2-4}{x-2} & ; x\neq 2\\ 2a^2+ax & ; x=2\end{cases}$ $g(x)=x+2$ $a=?$ -۱۰

$x=2 \rightsquigarrow f(2)=2a^2+2a=4 \rightsquigarrow (2a^2+2a-4=0)\div 2 \rightsquigarrow a^2+a-2=0$

$(a+2)(a-1)=0 \rightsquigarrow a=1$ ی -2

MIRNOTE