آنیا شکرچی

$$f(x)=\begin{cases} x^2+2x & ; x \geq a \\ 9x-4 & x \leq a \end{cases}$$ ـ ①

$a^2+2a = a^2-4 \qquad 2a=-4$

$\boxed{a=-2}$

$f(x)=\frac{x^2+a}{2x-b} \qquad g(x)=2x+b \qquad f(1)=?$ ـ ②

$\frac{2^2+a}{4-b}=4+b=3 \qquad \frac{4+a}{4-(-1)}=3 \qquad 12=4+a$

$\boxed{b=-1} \qquad \frac{4+a}{5} \qquad \boxed{a=11}$

$f(1)=\frac{1^2+11}{2\times1-(-1)}=\frac{12}{3}=\boxed{4}$

$f(x)=\frac{4x+1}{2x^2+ax+b}$ ins $\rightarrow R-\{-1,4\} \qquad f(1)=?$ ـ ③

$2(x+1)(x-4)=(2x+2)(x-4)=2x^2-8x+2x-8=2x^2-6x-8$

$2x^2+ax+b=2x^2-6x-8 \rightarrow a=-6$

$b=-8$

$f(1)=\frac{4+1}{2+(-6)+(-8)}=\boxed{\frac{5}{-12}}$

$f(x)=\frac{x^3-\sqrt{x}}{-4x^2+ax+b}$ ins: $R-\{-1\} \qquad a+b=?$ ـ ④

$-4(x+1)^2=-4(x^2+1+2x)=-4x^2-4-8x \qquad \begin{matrix} b=-4 \\ a=-8 \end{matrix} \Big\} a+b=\boxed{-12}$

۵)

$$f(x) = \frac{2x}{(x-1)(x^2+mx+1)}$$

دامنه : $R - \{1\}$

x^2+mx+1 → ریشه ندارد → $\Delta < 0$

$m^2 - 4 < 0 \qquad m^2 < 4 \rightarrow -2 < m < 2 \implies [-2, 2)$

$\Delta = 0 \rightarrow m = -2$

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۶)

$$f(x) = \sqrt{4 - \frac{1}{x^2}}$$

$-\frac{1}{2} \quad 0 \quad \frac{1}{2}$

$+ \quad - \quad - \quad +$

$4 - \frac{1}{x^2} \geq 0 \qquad \frac{+1}{x^2} \leq 4$

$\frac{1}{x^2} - 4 \leq 0 \rightarrow \frac{-4x^2+1}{x^2} \leq 0$ ← $-\frac{1}{2}$ و $\frac{1}{2}$ ، x : صفر $\longrightarrow (-\infty, -\frac{1}{2}] \cup [\frac{1}{2}, +\infty)$

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۷)

$$f(x) = \sqrt{mx^2 + 2mx + 1}$$

دامنه : $\mathbb{R}$

$\underset{a}{m}x^2 + \underset{b}{2m}x + \underset{c}{1} \geq 0$

$a > 0 \rightarrow m > 0$

$\Delta < 0$

$4m^2 - 4m < 0 \rightarrow m(4m-4)$ (ریشه‌ها: 0 و 1)

$+ \quad 0 \quad - \quad 1 \quad +$

$m = [0, 1]$

$m = 0 \rightarrow 1$

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۸)

$$f(x) \begin{cases} \dfrac{4x^2-1}{2x-1}, & x \neq \frac{1}{2} \\ 4a + k, & x = \frac{1}{2} \end{cases}$$

(زیر $4a + k$ نوشته شده: $2+k$)

$g(x) = 2x + 1$

$\underset{\frac{1}{2}}{a} + \underset{0}{k} = ? = \frac{1}{2}$

$2x - 1 \neq 0$

$2x \neq 1$

$x \neq \frac{1}{2}$

$2 \times \frac{1}{2} + 1 = 4 \times \frac{1}{2} + k$

$2 = 2 + k \longrightarrow k = 0$

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۹)

$$f(x) \begin{cases} \dfrac{9x^2-4}{3x+2} & x \neq -\frac{2}{3} \\ 3ax + 2 & x = -\frac{2}{3} \end{cases}$$

$g(x) = 3x + b$

$a - b = ?$

$f(x) = g(x) \longrightarrow \frac{9x^2-4}{3x+2} = 3x + b \rightarrow 9x^2 - 4 = (3x+b)(3x+2) \longrightarrow b = -2$

$3 \times \frac{-2}{3} \times a + 2 = 3 \times \frac{-2}{3} - 2$

$a - b = 3 - (-2) = 5$

$-2a + 2 = -2 - 2 = -4$

$-2a = -6 \longrightarrow a = 3$

$$f(x) = \begin{cases} \frac{x^2-4}{x-2} & , x \neq 2 \\ 2a^2 + ax & , x = 2 \end{cases}$$

$$g(x) = x + 2$$

$f(x) = g(x)$

$2a^2 + 2a = x^2 + 2 = 4$

$2a(a+1) = 4 \longrightarrow a = 1, -2$