۲۰

آنیا قندچی

(۱) ص

$$f(x) = \begin{cases} x^2+2x & ; x \geq a \\ ax-4 & x \leq a \end{cases}$$

$$a^2+2a = a^2-4 \qquad 2a=-4 \qquad \boxed{a=-2}$$

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(۲) ص

$$f(x) = \frac{x^2+a}{2x-b} \qquad g(x) = 2x+b \qquad f(1)=?$$

$$\frac{2^2+a}{4-b} = \qquad 4+b=3 \qquad \boxed{b=-1}$$

$$\frac{4+a}{4-(-1)} = 3 \qquad (4-(-1) = 5) \qquad 15 = 4+a \qquad \boxed{a=11}$$

$$f(1) = \frac{1^2+11}{2\times1-(-1)} = \frac{12}{3} = \boxed{4}$$

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(۳) ص

$$f(x) = \frac{4x+1}{2x^2+ax+b} \quad \text{دامنه}: R-\{-1,4\} \qquad f(1)=?$$

$$2(x+1)(x-4) = (2x+2)(x-4) = 2x^2-8x+2x-8 = 2x^2-6x-8$$

$$2x^2+ax+b = 2x^2-6x-8 \rightarrow a=-6,\ b=-8$$

$$f(1) = \frac{4+1}{2+(-6)+(-8)} = \boxed{\frac{5}{-12}}$$

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(۴) ص

$$f(x) = \frac{x^3-\sqrt{4}}{-4x^2+ax+b} \quad \text{دامنه}: R-\{-1\} \qquad a+b=?$$

$$-4(x+1)^2 = -4(x^2+1+2x) = -4x^2-4-8x$$

$$\left.\begin{matrix} b=-4 \\ a=-8 \end{matrix}\right\} a+b = \boxed{-12}$$

۶- $f(x)=\frac{2x}{(x-1)(x^2+mx+1)}$ دامنه: $R-\{1\}$

x^2+mx+1 ← نباید ریشه داشته باشد ← $\Delta<0$

$m^2-4<0 \quad m^2<4 \rightarrow -2<m<2 \Rightarrow [-2,2)$

$\Delta=0 \rightarrow m=-2$

۷- $f(x)=\sqrt{4-\frac{1}{x^2}}$

$4-\frac{1}{x^2}\geq 0 \quad \frac{+1}{x^2}<4$

$\frac{1}{x^2}-4\leq 0 \rightarrow \frac{-4x^2+1}{x}\leq 0 \rightarrow (-\infty,-\frac{1}{2}]\cup[\frac{1}{2},+\infty)$

۸- $f(x)=\sqrt{mx^2+2mx+1}$ دامنه: R

$mx^2+2mx+1\geq 0$

$a>0 \rightarrow m>0$

$\Delta\leq 0$

$4m^2-4m\leq 0 \rightarrow m(4m-4)$ $m=[0,1]$

$m=0 \rightarrow 1$

۹- $f(x)\begin{cases}\frac{4x^2-1}{2x-1} & x\neq \frac{1}{2} \\ 4x+k & x=\frac{1}{2}\end{cases}$ $g(x)=2x+1$

$2x-1\neq 0$

$2x\neq 1$

$x\neq\frac{1}{2}$

$4\times\frac{1}{2}+1=2\times\frac{1}{2}+k$

$2=1+k \rightarrow k=0$

$\frac{a}{b}+\frac{k}{b}=?=\frac{1}{2}$

۱۰- $f(x)\begin{cases}\frac{9x^2-4}{3x+2} & x\neq -\frac{2}{3} \\ 3ax+2 & x=-\frac{2}{3}\end{cases}$ $g(x)=3x+b$

$a-b=?$

$f(x)=g(x) \rightarrow \frac{9x^2-4}{3x+2}=3x+b \rightarrow 9x^2-4=(3x+b)(3x+2) \rightarrow b=-2$

$3\times\frac{-2}{3}\times a+2=3\times\frac{-2}{3}-2$

$-2a+2=-2-2=-4$

$-2a=-6 \rightarrow a=3$

$a-b=3-(-2)=5$

$$f(x)=\begin{cases}\dfrac{x^2-4}{x-2} & , x\neq 2\\ 2a^2+ax & , x=2\end{cases}$$

$g(x)=x+2$

(10) (2)

$f(x)=g(x)$

$2a^2+2a=x+2=4$

$2a(a+1)=4 \longrightarrow a=1, -2.$