شماره‌ی تکلیف ۲۳ — به نام خدا — زهرا سرابی

۲۰

$1 = -\frac{1}{2} \times 9 - 3 + \frac{17}{2}$

$-\frac{9}{2} - \frac{4}{2} + \frac{17}{2}$

(۱ $y = ax^2 + bx + c \quad , A(-1, 9)$

$y = a(x - x_s)^2 + y_s \rightarrow 1 = a(x+1)^2 + 9 \rightarrow 1 = a(\cancel{3+1})^2 + 9 \rightarrow -8 = 16a \rightarrow a = -\frac{1}{2}$ $\quad \boxed{y = -\frac{1}{2}(x+1)^2 + 9} \rightarrow -\frac{1}{2}x^2 - x + \frac{17}{2}$

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(۲ $2x^2 + mx + m + 6 = 0$

$\Delta > 0 \rightarrow m^2 - 4(2)(m+6) \rightarrow m^2 - 8m - 48 > 0 \rightarrow (m-12)(m+4) > 0$, $\quad 12 \quad -4$

$\frac{c}{a} > 0 \qquad \frac{-b}{a} > 0 \rightarrow \frac{-m}{2} > 0 \rightarrow m < 0$

$\frac{m+6}{2} > 0 \rightarrow m > -6$

$-6 \quad -4 \quad 0 \quad 12$

$\boxed{(-6, -4)}$

ج: $(-\infty, -4) \cup (12, +\infty)$

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(۳ $3x^2 + (2m-1)x + 2 - m = 0$

$-\frac{b}{a} = \frac{a}{c} \rightarrow \frac{1-2m}{3} = \frac{3}{2-m} \rightarrow 9 = (1-2m)(2-m) \rightarrow 2 - m - 4m + 2m^2 \rightarrow 2m^2 - 5m + 2 - 9 = 0 \rightarrow 2m^2 - 5m - 7 = 0$

پاسخ: $\boxed{-\frac{c}{a} = \frac{7}{2}}$ $\quad (-1)$ قابل قبول نیست.

$3x^2 - 3x + 3 = 0$

$9 - 4(3)(3) < 0$

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(۴ $x = x^2 - 4$ $\qquad \alpha \; x_1 \quad \beta \; x_2$

$\begin{cases} x^2 - x - 4 = 0 \\ \alpha + \beta = S = -\frac{b}{a} = 1 \\ \alpha \cdot \beta = P = \frac{c}{a} = -4 \end{cases}$

$\frac{\beta+\alpha}{\alpha\beta} = \frac{1}{-4} = -\frac{1}{4}$

$\beta^3 + \frac{1}{\alpha} + \alpha^3 + \frac{1}{\beta} \rightarrow \alpha^3 + \beta^3 + \frac{1}{\alpha} + \frac{1}{\beta}$

$S^3 - 3SP \rightarrow 1 - 3(1)(-4) = 13 - \frac{1}{4} = \frac{51}{4} = S$

$(\beta^3 + \frac{1}{\alpha}) \times (\alpha^3 + \frac{1}{\beta})$

$(\alpha\beta)^3 + \beta^3(\frac{1}{\beta}) + \frac{1}{\alpha}(\alpha^3) + \frac{1}{\alpha\beta} = (\alpha\beta)^3 + \beta^2 + \alpha^2 + \frac{1}{\alpha\beta}$

$(-4)^3 + S^2 - 2P + \frac{1}{-4}$

$P = -64 + \cancel{1 - 2(-4)} - \frac{1}{4} = -55 - \frac{1}{4} = -\frac{221}{4}$

$x^2 - Sx + P$

$x^2 - (\frac{51}{4})x - \frac{221}{4} \rightarrow \boxed{4x^2 - 51x - 221 = 0}$

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(۵ $\left(\sqrt[3]{x^2} + \frac{1}{\sqrt[3]{x^2}} + 1\right)\left(\sqrt[3]{x^2} - 1\right) = 2\sqrt[3]{x}$ $\qquad \sqrt[3]{x} = t$

$x_1 + x_2 = \frac{-b}{a} \rightarrow \frac{2}{1} = \boxed{2}$ پاسخ

$(t^2 + \frac{1}{t^2} + 1)(t^2 - 1) = 2t$

$(\frac{t^4 + 1 + t^2}{t^2})(t^2 - 1) = \frac{t^6 - 1}{t^2} = 2t \rightarrow t^6 - 2t^3 - 1 = 0$

$(x^{\frac{1}{3}})^6 - 2(x^{\frac{1}{3}})^3 - 1 = 0 \rightarrow x^2 - 2x - 1 = 0$

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(۶ $3x^2 - ax + 4 = 0$

$\alpha = 3\beta$

$\alpha \cdot \beta = \frac{c}{a} = \frac{4}{3} = 3\beta^2 \rightarrow 4 = 9\beta^2 \rightarrow \frac{4}{9} = \beta^2 \rightarrow \beta = \pm\frac{2}{3}$

$\beta = \frac{2}{3} \rightarrow \alpha = 2$

$\beta = -\frac{2}{3} \rightarrow \alpha = -2$

$\alpha + \beta = -\frac{b}{a}$: $\frac{8}{3} = \frac{a}{3} \rightarrow a = 8$; $-\frac{8}{3} = \frac{a}{3} \rightarrow a = -8$

پاسخ: $|a_1 - a_2| = |8 - (-8)| = 16$

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(۷ $y = ax^2 + (3 + 2a)x$

$a > 0$

$\Delta \geq 0 \rightarrow (3+2a)^2 - 4(a)(0) = 9 + 4a^2 + 12a \geq 0 \rightarrow \Delta = 144 - 4(4)(9) = 0$

$a = -\frac{b}{2a} = -\frac{12}{8} = \boxed{-\frac{3}{2}}$

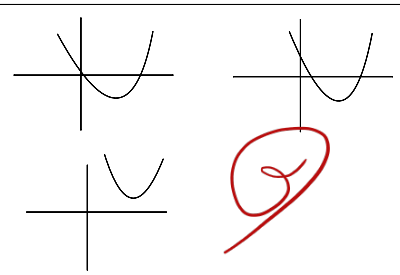
$\frac{c}{a} \geq 0 \rightarrow 0$

$-\frac{b}{a} > 0 \rightarrow \frac{-3-2a}{a} > 0$, $-\frac{3}{2}$

$-\frac{3}{2} < a < 0$

$-\frac{3}{2} \quad 0$

$\boxed{\text{هیچ مقدار } a}$



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$y=-x^{۲}-۲x+b$ شرن $x=-\frac{b}{۲a}=\frac{۲}{-۲}=\frac{-a}{۲}\rightarrow ۴=۲a\rightarrow a=۲$ $-x^{۲}-۲x+b=۱$ (۸

$y=x^{۲}+ax-۲$ $-x^{۲}-۲x+b-۱=0$

$x^{۲}+۲x-۲=۱\rightarrow x^{۲}+۲x-۳=0\rightarrow ۱=x$ $x=۱\rightarrow -۱-۲+b-۱=0\rightarrow \boxed{b=۴}$

$\rightarrow -۳=\frac{c}{a}=x$ $x=-۳\rightarrow -۹+۶+b-۱=0\rightarrow \boxed{b=۴}$

پاسخ $ab=۴\times۲=\boxed{۸}$

$۲x^{۲}-ax+b=۰$ $۲ax^{۲}+ax-۶=۰$ (۹

α,β $\underbrace{\alpha+\frac{1}{۲},\beta+\frac{1}{۲}}$

$S:-\frac{b}{a}=\frac{+a}{۲}=\alpha+\beta\rightarrow S=\frac{-1}{۲}$ $S=\alpha+\beta+1=\frac{a}{۲}+1=\frac{-a}{۲}\rightarrow 1=\frac{-\cancel{۲}a}{\cancel{۲}}\rightarrow \boxed{a=-1}$

$P=\frac{c}{a}=\frac{b}{۲}=\alpha\cdot\beta$ $\left[\frac{ab}{۴}\right]=\left[\frac{-۶}{۴}\right]=-۲$

$P=\underbrace{\alpha\beta}+\underbrace{\frac{\alpha}{۲}+\frac{\beta}{۲}}_{\frac{\alpha+\beta}{۲}}+\frac{1}{۴}=\frac{-۶}{۲a}=\frac{-۳}{a}\rightarrow\frac{-۳}{-1}=۳\rightarrow\frac{b}{۲}-\frac{1}{۴}+\frac{1}{۴}=۳\rightarrow\boxed{b=۶}$

$\frac{\alpha+\beta}{۲}\rightarrow\left(\frac{-\frac{1}{۲}}{\frac{۲}{1}}\right)=-\frac{1}{۴}$

$x^{۲}+۶x+m=۰\begin{cases}\alpha\\ \beta\end{cases}$ $\alpha+\beta=-۶$ $\cancel{\alpha}+\beta=-۶$ (۱۰

$x^{۲}+۲x-۳m=۰\begin{cases}\alpha\\ \beta'\end{cases}$ $\alpha+\beta'=-۲$ $\times -$ $-\cancel{\alpha}-\beta'=۲$

$\beta-\beta'=-۴$