هلیا عرب

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1) $9 = 4a - 2b + c$ $\qquad -\dfrac{b}{2a} = -1$

$1 = 9a + 3b + c$ $\qquad -b = -2a \Rightarrow b = 2a$

$$2-1 \begin{cases} 9 = 4a - 4a + c \\ 1 = 9a + 6a + c \end{cases} \Rightarrow -8 = 16a \Rightarrow a = -\frac{1}{2} \rightarrow b = 2 \times -\frac{1}{2} = -1$$

$15a$

معادله : $-\dfrac{1}{2}x^2 - x + \dfrac{17}{2}$

$-a + c = 9$

$c = 9 - \dfrac{1}{2} \Rightarrow c = \dfrac{17}{2}$

2) $2x^2 + mx + m + 6 = 0$

$\Delta > 0 \Rightarrow b^2 - 4ac > 0$

$m^2 - 4(2)(m+6) > 0$

$m^2 - 8m - 48 > 0 \Rightarrow m > 12$ (3)

$(m-12)(m+4)$ $\qquad m < -4$ (4)

$S = -\dfrac{b}{a} > 0 \Rightarrow -\dfrac{m}{2} > 0$ $\quad m < 0$ (1)

$P = \dfrac{c}{a} > 0 \Rightarrow \dfrac{m+6}{2} > 0 \Rightarrow m + 6 > 0$

$m > -6$ (2)

$\boxed{-6 < m < -4}$

3) $-\dfrac{b}{a} = \dfrac{1}{\frac{c}{a}} \Rightarrow -\dfrac{b}{a} = \dfrac{a}{c} \Rightarrow \dfrac{-2m+1}{2} = \dfrac{3}{2-m} \Rightarrow -4m + 2 + 2m^2 - m = 9$

$a + c = b$ $\qquad 2m^2 - 5m - 7 = 0$

$\Delta < 0 \leftarrow 2x^2 - x + 5 = 0 \leftarrow$ غ ق ق $\leftarrow m = -1$

$\Delta > 0 \leftarrow m = -\dfrac{c}{a} \Rightarrow \boxed{\dfrac{7}{2}}$

4) $x = x^2 - 4 \Rightarrow x^2 - 4 - x = 0$

$x_1 + x_2 = 1$

$x_1 \cdot x_2 = -4$

$S = x_2^2 + \dfrac{1}{x_1} + x_1^2 + \dfrac{1}{x_2} = \underbrace{x_1^2 + x_2^2}_{S^2 - 2PS} + \underbrace{\dfrac{1}{x_1} + \dfrac{1}{x_2}}_{\frac{x_1 + x_2}{x_1 \times x_2}}$

$\downarrow$

$1 + 12 - \dfrac{1}{4} = \dfrac{51}{4}$

$P = \left(x_2^2 + \dfrac{1}{x_1}\right)\left(x_1^2 + \dfrac{1}{x_2}\right) =$

$P^2 + \underbrace{x_2^3 + x_1^3}_{S^3 - 3SP} + \dfrac{1}{P} =$

$-64 + 9 - \dfrac{1}{4} = -\dfrac{231}{4}$

$y \overset{\times 4}{=} \left(x^2 - \dfrac{51}{4}x - \dfrac{231}{4}\right)$

$0 = 4x^2 - 51x + 231$

5) $\sqrt[3]{x} = t \Rightarrow \left(t^2 + \dfrac{1}{t^2} + 1\right)(t^2 + 1) = 2t \Rightarrow \cancel{t^2(t^2-1) + \dfrac{1}{t^2}(t^2-1) + 1(t^2-1)}$

و $x = t^3$

$t^4 - t^2$ $\qquad 1 - \dfrac{1}{t^2}$ $\qquad t^2 - 1$

$t^6 - 2t^3 - 1 = 0 \leftarrow t^6 - 1 = 2t^3 \leftarrow t^2 \times \left(t^4 - \dfrac{1}{t^2}\right) \leftarrow t^4 - \dfrac{1}{t^2}$

$x = \dfrac{2 \pm \sqrt{8}}{2} \Rightarrow x = 1 \pm \sqrt{2}$

$x_1 + x_2 = 1 + \cancel{\sqrt{2}} + 1 - \cancel{\sqrt{2}} = 2$

۶) $\alpha = 3\beta \Rightarrow 4\beta = \frac{a}{4} \Rightarrow a = 16\beta$ $\qquad 8-(-8) = 16$

$\alpha\cdot\beta = \frac{4}{3} \Rightarrow 3\beta^2 = \frac{4}{3} \Rightarrow \beta^2 = \frac{4}{9} \Rightarrow \beta_1 = \frac{2}{3}\times 12 = 8$

$\beta_2 = \frac{-2}{3}\times 12 = -8$

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۷) $y = x(ax + (3+2a)) = 0$

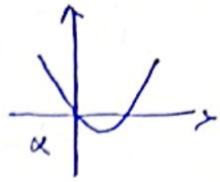
$\to 0$

$ax + (3+2a) = 0 \Rightarrow x = \frac{-3-2a}{a} > 0$

بزرگتر از جمع ریشه ها

$a>0$

$y = -x \Rightarrow a \neq 0$ $\Rightarrow \beta+\alpha = \frac{-b}{a} > 0$



$-\frac{3}{2}$		0	
$-$	$+$		$-$

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۸) $y = x^2 + ax - 2$, $y = -x^2 - 2x + b$

طول رأس $= \frac{-b}{2a} = \frac{-a}{2} \Rightarrow x_2 = \frac{-(-2)}{2(-1)} = \frac{2}{-2} = -1$

$\frac{-a}{2} = -1 \Rightarrow a = 2$

$x^2 + 2x - 2 = 1 \to x^2 + 2x - 3 = 0 \Rightarrow (x+3)(x-1) = 0$, $x = 1$, $x = -3$

$-x^2 - 2x + b \xrightarrow{x=1} y = -1 - 2 + b = b-3$

$x = -3 \Rightarrow y = -9 + 6 + b = b - 3$ $\Big\} \to b - 3 = 1 \Rightarrow b = 4$

$a \cdot b = 8$

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۹) $2x^2 - ax + b = 0$ / $2x^2 + ax - 6 = 0$

x_1, x_2 $\qquad y_1, y_2$

$S = \frac{a}{2}$ $\qquad S = -\frac{a}{2a} = -\frac{1}{2}$

$P = \frac{b}{2}$ $\qquad P = -\frac{3}{a}$

$x_1 + x_2 = y_1 + y_2 + 2\times\frac{1}{2} \to \frac{a}{2} = -\frac{1}{2} + 1$

$a = 1$

$x_1 x_2 = (y_1 + \frac{1}{2})(y_2 + \frac{1}{2}) \to$

$y_1 \times y_2 + \frac{1}{2}(y_1 + y_2) + \frac{1}{4}$

$-3 + \frac{1}{2}(-\frac{1}{2}) + \frac{1}{4} = -3$

$x_1 \times x_2 = -3 = \frac{b}{2} \Rightarrow b = -6$

$\left[\frac{-6\times 1}{4}\right] = -2$

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۱۰) $x^2 + 6x + m = 0$ $\quad \alpha, \beta$

$x^2 + 2x - 3m = 0$ $\quad \alpha, \beta'$

$\alpha + \beta = -6$ $\qquad (x-1)\alpha + \beta' = -2$

$\alpha + \beta = -6$

$-\alpha + \beta' = 2$

$\beta - \beta' = -4$