Subject:
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۱۴۰۱، ۲۳

تکلیف سری ۲۳ — دهم دختر A — نازنین زهرا شفیعی

سوال ۱:

$y = ax^2 + bx + c \Rightarrow y = ax^2 + 2ax + a - 9$

$y = \frac{a}{4}x^2 + \frac{a}{2}x - 9/8$

$-\frac{\Delta}{4a} = -\frac{4a^2 - 4ac}{4a} = \frac{4a(a-c)}{4a} = 9 \quad a - c = 9 \Rightarrow c = a - 9$

$(2,1) \rightarrow 1 = 9a + 9a + 9 - 9 \Rightarrow 18a = 1 \Rightarrow a = \frac{1}{18} = \frac{9}{8}$

۵، ۵

سوال ۲:

$2x^2 + mx + m + 4 = 0$

$\Delta > 0 \Rightarrow m^2 - 8m - 48 > 0 \quad (m+12)(m+4) > 0 \quad m > 12, m < -4$

$P > 0 \Rightarrow \frac{c}{a} > 0 \quad \frac{m+4}{2} > 0 \quad m > -4$

$S > 0 \Rightarrow \frac{-m}{2} > 0 \quad m < 0$

$-4 < m < 0$

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سوال ۳:

$2x^2 + (2m-1)x + 2 - m = 0$

$m = 7/2$

$S = \alpha + \beta = -\frac{b}{a} = \frac{1-2m}{2} \Rightarrow \frac{1-2m}{2} = \frac{3}{2-m}$

$P = \frac{c}{a} = \frac{2-m}{2}$

$(1-2m)(2-m) = 9$

$2m^2 - 5m + 2 = 9$

$m^2 - 5m - 14 = 0$

$(m-7)(m+2) = 0$

$(2m-7)(m+1) = 0$

$m = 7/2 \quad m = -1$

$S = \frac{1}{P} \Rightarrow 4m + 14 > 0$

$\Delta > 0 \rightarrow 4x^2 + 4x - \frac{9}{2} = 0$

$m = -1 \rightarrow 2x^2 - 2x + 1 = 0 \quad \Delta < 0$

$9 - 12 < 0$

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سوال ۴:

$x = x^2 - 4 \Rightarrow x^2 - x - 4 = 0 \quad x_1 + x_2 = 1 \quad x_1 \cdot x_2 = -4$

ریشه‌های جدید: $y_1 = x_1^3 + \frac{1}{x_1}$

$y_2 = x_2^3 + \frac{1}{x_2}$

$\Rightarrow y^2 - Sy + Py = 0 \Rightarrow (x_1^3 + x_2^3) = (x_1 + x_2) - 3x_1x_2(x_1 + x_2)$

$1 + 12 = 13$

$P_y = \left(x_1^3 + \frac{1}{x_1}\right)\left(x_2^3 + \frac{1}{x_2}\right) \Rightarrow P_y = -\frac{41}{4}$

$\frac{1}{x_1} + \frac{1}{x_2} = -\frac{1}{4} \Rightarrow S_y = \frac{41}{4}$

$x_1^3 x_2^3 + \frac{x_2^3}{x_1} + \frac{x_1^3}{x_2} + \frac{1}{x_1 x_2}$ جبر

معادله جدید: $x^2 - \frac{41}{4}x - \frac{41}{4} = 0 \Rightarrow 4x^2 - 41x - 41 = 0$

سوال ۵:

$\left(\sqrt[3]{x^2} + \frac{1}{\sqrt[3]{x^2}} + 1\right)\left(\sqrt[3]{x^2} - 1\right) = 2\sqrt{x}$

$\left(\frac{\sqrt[3]{x^4} + 1 + \sqrt[3]{x^2}}{\sqrt[3]{x^2}}\right)\left(\sqrt[3]{x^2} - 1\right) = 2\sqrt{x}$

$x^2 - 1 = 2x \Rightarrow x^2 - 2x - 1 = 0$

$\alpha + \beta = -\frac{b}{a} = 2$

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پیگیو

سوال ۴: (۲.۶)

$3x^2 - ax + 4 = 0$

$S = 4\alpha = \frac{a}{3} \Rightarrow \alpha = \frac{a}{12}$

$P = 3\alpha^2 = \frac{4}{3} \Rightarrow \alpha^2 = \frac{4}{9} \Rightarrow \frac{a^2}{144} = \frac{4}{9} \Rightarrow a^2 = 64 \Rightarrow a = \pm 8$ تفاضل $\Rightarrow$ $\boxed{8-(-8)=16}$

سوال ۵: (۵)

$y = ax^2 + (3+2a)x$

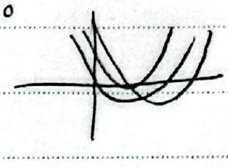
$x<0 , y<0$ ✗ $\Rightarrow x<0 \Rightarrow y \geq 0$

$ax^2 + (3+2a)x \geq 0 \Rightarrow x(ax+3+2a) \geq 0 \Rightarrow ax + (3+2a) \leq 0$

$a>0 \overset{=1}{\Rightarrow} x+5 \leq 0 \Rightarrow x \leq -5$ ✓ ~~$\boxed{a \geq 0}$~~

$a=0 \Rightarrow 3 \leq 0$ تناقض

$a<0 \overset{-1}{\Rightarrow} -x+1 \geq 0 \Rightarrow x \geq 1$ ✗



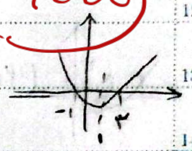
سوال ۸: (۵،۱۸۵)

$y = -x^2 - 2x + b \quad , \quad y = x^2 + ax - 2 \Rightarrow x^2 - 2x - 2 = y$

محور تقارن: $x = -1$ (۱) ، محور تقارن: $-\frac{a}{2}$ ، $-\frac{a}{2} = 1 \Rightarrow \boxed{a=-2}$ ✓

فاصله از محور تقارن: $x_1 - 1 , x_2 + 1$ جایگذاری $\Rightarrow$ x کلی.

$y=1$ ، $x^2 - 2x - 3 = 0$ ، $(x-3)(x+1) = 0$ ، $x=3 , x=-1$



$-(1+x_1) - 2(1+x_1) + b = -x_1^2 - 4x_1 - 3 + b = y$

$y=1 \Rightarrow 1 = -x_1^2 - 4x_1 - 3 + b$

$b = x^2 + 4x + 4 \Rightarrow b = (x+2)^2 \overset{(1)}{\Rightarrow}$ ~~$\boxed{b=16}$~~

فاصله از محور تقارن (۱) $= 2$ ~~$\boxed{ab=-32}$~~

سوال ۹: (۲)

$3x^2 - ax + b = 0 \quad , \quad 2ax^2 + ax - 6 = 0$

$S = \frac{a}{3}$ ، $\beta = (p + \frac{1}{2})(q + \frac{1}{2})$ ، $S = -\frac{1}{2}$

$P = \frac{b}{3}$ ، $P = -\frac{3}{a}$

مجموع ریشه‌ها $\Rightarrow -\frac{1}{2} + 1 = \frac{a}{2} = \boxed{a=1}$ $\Rightarrow \left[\frac{ab}{4}\right] = \left[\frac{-6}{4}\right] = \left[\frac{-3}{2}\right] = [-1.5] = -2$

ضرب ریشه‌ها $\Rightarrow -3 + \frac{1}{4} + \frac{1}{4} \;\; -3 = \frac{b}{2} = \boxed{b=-6}$

سوال ۱۰: (۵)

$x^2 + 4x + m = 0 \quad , \quad x^2 + 2x - 3m = 0 \quad\quad x^2 + 2x + 15 = 0$

$\alpha^2 + 4\alpha + m = 0 \quad\quad \alpha^2 + 2\alpha - 3m = 0 \Rightarrow m = \frac{\alpha^2 + 2\alpha}{3} \Rightarrow \frac{25-10}{3} = 5 = \boxed{m}$

$m = -(\alpha^2 + 4\alpha)$ تساوی $\Rightarrow -(\alpha^2 + 4\alpha) = \frac{\alpha^2 + 2\alpha}{3} \Rightarrow -3\alpha^2 - 12\alpha = \alpha^2 + 2\alpha$

جایگذاری $\alpha = -5$: $-25 + 30 \Rightarrow \boxed{m=5}$ $\quad\quad -4\alpha(\alpha+5) \quad -4\alpha^2 - 20\alpha = 0$

$x^2 + 4x - 5 = 0 \quad\quad \alpha^2 + 5\alpha = 0$

$S_1 = -\frac{b}{a} = -\frac{4}{1} = -4 \Rightarrow -4 = -5 + (1)$ ریشه دیگر $\quad\quad \alpha \neq 0$ فرض می‌کنیم $\leftarrow \alpha(\alpha+5) = 0$

$S_2 = -2 \Rightarrow -2 = -5 + (3)$ ریشه دیگر $\quad$ اختلاف $\Rightarrow 3 + 1 = (4)$ اختلاف دو ریشه دیگر $\quad\quad \boxed{\alpha = -5}$