ایلار علیاری مثلثات ۲۰ شمارهٔ دانش : ۱۹

۱) ب) الف) ۱۰:۵۴

$a=|5.5m-30h|$

$a=|5.5\times54-30\times10| = 207$

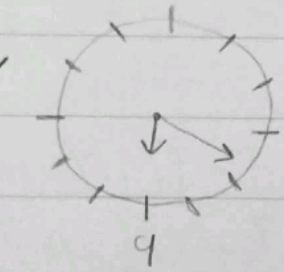
زاویه $153 = 207 - 360$

$\frac{54}{2}=27$

$30-(4\times6)=6$ $\quad 120+6+27=153$ زاویه

$4\times30=120$

۲) ب) الف) ۹:۱۸



$a=|5.5m-30h|$

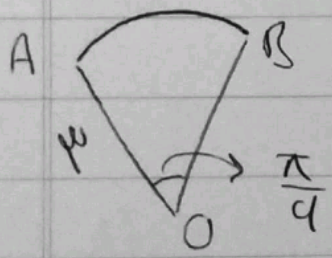
$a=|5.5\times18-30\times9| = 81°$ زاویه

$\frac{18}{2}=9$

$2\times30=60$ $\quad 60+9+12=81$ زاویه

$30-(3\times6)=12$

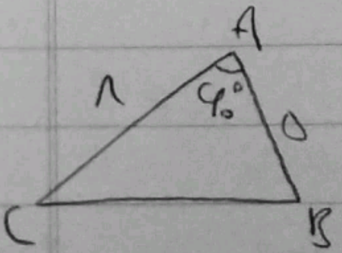
۳)



الف) $S_{OAB}=\frac{a}{2}R^2=\frac{\pi}{12}\times9=\frac{3\pi}{4}$

ب) $P_{OAB}=2R+\widehat{AB}=2\times3+\frac{\pi}{2}=6+\frac{\pi}{2}$

$\widehat{AB}=aR=3\times\frac{\pi}{6}=\frac{\pi}{2}$

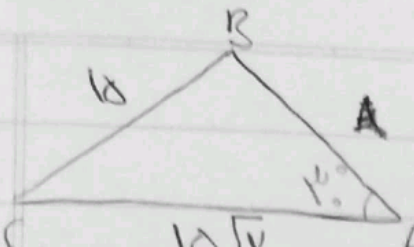


۴) الف) $S_{ABC}=\frac12 ab\sin a=\frac12\times5\times8\times\sin60°=10\sqrt3$ $\left(\frac{\sqrt3}{2}\right)$

ب) $P_{ABC}=AB+BC+AC=5+7+8=20$

$BC^2=a^2+b^2-2ab\cos A$

$BC^2=25+64-80\times\frac12=49 \Rightarrow BC=7$



$\hat{B}+\hat{C}=۱۵۰^\circ \Rightarrow \hat{A}=۱۸۰-۱۵۰=۳۰^\circ$

$\frac{a}{\sin A}=\frac{b}{\sin B} \Rightarrow \frac{۱۰۰}{\frac{۱}{۲}}=\frac{۱۵\sqrt{۲}}{\sin B}$

$\sin B=\frac{\sqrt{۲}}{۲}$

$B=۴۵^\circ \Rightarrow \hat{B}+\hat{C}=۱۵۰^\circ$

$C=۱۰۵^\circ$

۱۵

$\hat{B}=\frac{۴۵}{۱۸۰}\pi=\frac{\pi}{۴}$

$\hat{C}=\frac{۱۰۵}{۱۸۰}\pi=\frac{۷\pi}{۱۲}$

۱۴

$$\frac{\tan(\pi-\alpha)+۲\tan(\pi+\alpha)}{\tan(۲\pi-\alpha)-\tan(۲\pi+\alpha)}=\frac{-\tan\alpha+۲\tan\alpha}{-\tan\alpha-\tan\alpha}=\frac{۲\tan\alpha}{-۲\tan\alpha}=-۱$$

۱۷

$$A=\frac{۲\tan۷۵^\circ+\tan۱۰۵^\circ}{۲\tan۱۴۵^\circ-\tan۲۵۵^\circ}=\frac{۲\tan(\frac{\pi}{۲}-۱۵)+\tan(\frac{\pi}{۲}+۱۵)}{۲\tan(\pi-۱۵)-\tan(۲\frac{\pi}{۲}-۱۵)}=$$

$$\frac{۲\cot۱۵^\circ-\cot۱۵^\circ}{-۲\tan۱۵^\circ-\cot۱۵^\circ}=\frac{\cot۱۵^\circ}{-۲\tan۱۵^\circ-\cot۱۵^\circ}=\frac{\frac{۱}{a}}{-۲a-\frac{۱}{a}}=\frac{\frac{۱}{a}}{\frac{-۲a^۲-۱}{a}}=\frac{-۱}{۲a^۲+۱}$$

$\tan\frac{\pi}{۱۲}=\tan۱۵^\circ=a \Rightarrow \cot۱۵^\circ=\frac{۱}{a}$

$\frac{\pi}{۱۲}\times\frac{۱۸۰}{\pi}=۱۵^\circ$

۱۸

$$\frac{\sin x+\cos x}{\sin x-\cos x}+\frac{\sin x-\cos x}{\sin x+\cos x}=۲$$

$\cos^۲x=۱-\sin^۲x$

$$\frac{\sin^۲x+\cos^۲x+۲\sin x\cos x+\sin^۲x+\cos^۲x-۲\sin x\cos x}{\sin^۲x-\cos^۲x}=\frac{۲}{\sin^۲x-\cos^۲x}=۲$$

$$\frac{۲}{۲\sin^۲x-۱}=۲$$

$۴\sin^۲x-۲=۲$

$\sin^۲x=\frac{۴}{۴} \Rightarrow \cos^۲x=\frac{۱}{۴}$

$\cos^۲x=۱-\sin^۲x$

$\tan^۲x=\frac{\sin^۲x}{\cos^۲x}=\frac{\frac{۳}{۴}}{\frac{۱}{۴}}=۳$

$$\frac{\sin^2 x - 2\cos^2 x + 1}{\sin^2 x + 2\cos^2 x - 1} = 4$$

$$\frac{\sin^2 x - 2 + 2\sin^2 x + 1}{\sin^2 x + 2 - 2\sin^2 x - 1} = 4 \quad \leftarrow \cos^2 x = 1 - \sin^2 x$$

$$3\sin^2 x - 1 = -4\sin^2 x + 4$$

$$7\sin^2 x = 5$$

$$\sin^2 x = \frac{5}{7} \Rightarrow \cos^2 x = \frac{2}{7}$$

$$\cos^2 x = 1 - \sin^2 x$$

$$\tan^2 x = \frac{\sin^2 x}{\cos^2 x} = \frac{\frac{5}{7}}{\frac{2}{7}} = \frac{5}{2} = 2{,}5$$

ب) $\cos(112{,}5^\circ)$

$$\cos^2 112{,}5^\circ = \frac{1 + \cos 225^\circ}{2} = \frac{2 + \sqrt{2}}{4} = \frac{2 + \sqrt{2}}{4}$$

$$\cos(112{,}5^\circ) = \underset{\text{ربع ثاني}}{-}\frac{\sqrt{2 + \sqrt{2}}}{2} = \frac{\sqrt{2 + \sqrt{2}}}{2}$$

ب) $\sin(67{,}5^\circ)$

$$\sin^2 67{,}5^\circ = \frac{1 - \cos 135^\circ}{2} = \frac{2 + \sqrt{2}}{4} = \frac{2 + \sqrt{2}}{4}$$

$$\sin(67{,}5^\circ) = \underset{\text{ربع أول}}{+}\frac{\sqrt{2 + \sqrt{2}}}{2} = \frac{\sqrt{2 + \sqrt{2}}}{2}$$