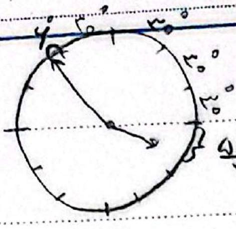
Subject:

Date :

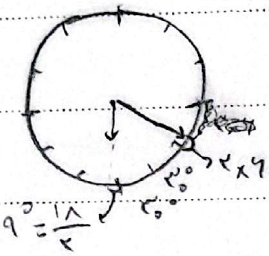
(۱)

الف) 

$\rightarrow 4(30) + 27 + 6 = 153^\circ$, $\frac{54}{2} = 27^\circ$

ب) $\alpha = |5/5M - 30H| = |(5/5 \times 54) - (30 \times 3)| = |297 - 90| = 207 \rightarrow 360 - 207 = 153^\circ$

(۲)

الف) 

$\rightarrow 2(30) + 12 + 9 = 81^\circ$, $2 \times 6 = 12^\circ$, $9^\circ = \frac{18}{2}$

ب) $\alpha = |(5/5 \times 18) - (30 \times 6)| = |99 - 180| = |-81| = 81^\circ$

(۳)

الف) $S = \frac{\alpha}{2} r^2 = \left(\frac{\frac{\pi}{6}}{2}\right) \times 9 = \frac{\pi}{12} \times 9 = \frac{3\pi}{4}$

ب) $P = 2r + \alpha r = 6 + \left(\frac{\pi}{6} \times 3\right) = 6 + \frac{\pi}{2}$

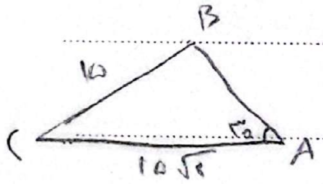
(۴)

الف) $S = \frac{1}{2} \times 5 \times 8 \times \sin 60 = 10\sqrt{3}$

ب) $BC = \sqrt{25 + 64 - \left(80 \times \frac{1}{2}\right)} = \sqrt{25 + 64 - 40}$

$= \sqrt{49} = 7 \rightarrow 7 + 8 + 5 = 20$

(۵)



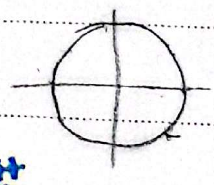
$\frac{a}{\sin A} = \frac{b}{\sin b} \rightarrow \frac{15}{\frac{1}{2}} = \frac{15\sqrt{2}}{\sin B} \rightarrow \sin B = \frac{\sqrt{2}}{2} \rightarrow B = 45^\circ$

$\rightarrow C = 180 - 75 = 105^\circ$

$\rightarrow B = \frac{\pi}{4}$, $C \Rightarrow \frac{Rad}{\pi} = \frac{105}{180} \rightarrow C = \frac{7\pi}{12}$

(۶)

$\frac{\tan(\pi - \alpha) + 3\tan(\pi + \alpha)}{\tan(2\pi - \alpha) - \tan(2\pi + \alpha)} = \frac{-\tan\alpha + 3\tan\alpha}{-\tan\alpha - \tan\alpha} = \frac{2\tan\alpha}{-2\tan\alpha} = \frac{2}{-2} = -1$



(۱۷) $\tan\frac{\pi}{12} = a \quad \left(\frac{\frac{\pi}{12}}{\frac{\pi}{1}} = \frac{1}{12} = \frac{?}{180} \to ? = 15 \to \cot 15 = \frac{1}{a}\right.$

$$\frac{2\tan 75^\circ + \tan 105^\circ}{3\tan 165^\circ - \tan 255^\circ} = \frac{2\overbrace{\tan(\frac{\pi}{2} - 15)}^{\cot 15^\circ} + \overbrace{\tan(\frac{\pi}{2} + 15)}^{-\cot 15^\circ}}{3\underbrace{\tan(\pi - 15)}_{-\tan 15} - \underbrace{\tan(\frac{3\pi}{2} - 15)}_{\cot 15}} = \frac{\frac{2}{a} - \frac{1}{a}}{-3a - \frac{1}{a}} = \frac{\frac{1}{a}}{\frac{-3a^2-1}{a}} = -\frac{1}{3a^2+1}$$

(۱۸)
$$\frac{\sin x + \cos x}{\sin x - \cos x} + \frac{\sin x - \cos x}{\sin x + \cos x} = \frac{\sin^2 x + \cos^2 x + 2\sin x\cos x + \sin^2 x + \cos^2 x - 2\sin x\cos x}{\sin^2 x - \cos^2 x}$$

$$= \frac{2}{1 - \cos^2 x - \cos^2 x} = 3 \to 2 = 3 - 6\cos^2 x \to 6\cos^2 x = 1 \to \cos^2 x = \frac{1}{6} \to \sin^2 x = \frac{5}{6}$$

$$\to \frac{\sin^2 x}{\cos^2 x} = \frac{\frac{5}{6}}{\frac{1}{6}} = \boxed{5} = \tan^2 x$$

(۱۹)
$$\frac{\sin^2 x - 2\cos^2 x + 1}{\sin^2 x + 2\cos^2 x - 1} = 4 \to \frac{1 - \cos^2 x - 2\cos^2 x + 1}{1 - \cos^2 x + 2\cos^2 x - 1} = \frac{2 - 3\cos^2 x}{\cos^2 x} = 4 \to 2 - 3\cos^2 x = 4\cos^2 x$$

$$\to 2 = 7\cos^2 x \to \cos^2 x = \frac{2}{7} \to \sin^2 x = \frac{5}{7} \to \tan^2 x = \frac{\sin^2 x}{\cos^2 x} = \frac{\frac{5}{7}}{\frac{2}{7}} = \frac{5}{2}$$

(۲۰)
الف) $\cos(22/5) \to \cos^2\alpha = \frac{1 + \cos 2\alpha}{2} \to \cos^2(22/5) = \frac{1 + \cos(45)}{2}$

$\to \frac{1 + \frac{\sqrt{2}}{2}}{2} = \frac{2 + \sqrt{2}}{4} \to \cos(22/5) = \frac{\sqrt{2 + \sqrt{2}}}{2} \to$ چون ناحیه اول است [illegible]

ب) $\sin(67/5) \to \sin^2\alpha = \frac{1 - \cos 2\alpha}{2} \to \sin^2(67/5) = \frac{1 - \cos(135)}{2}$

$= \frac{1 + \frac{\sqrt{2}}{2}}{2} = \frac{2 + \sqrt{2}}{4} \to \sin\alpha = \frac{\sqrt{2 + \sqrt{2}}}{2} \to$ چون ناحیه اول است [illegible]