- کیسا رادی ، ۱۸ -

۱-

الف) $\tan\frac{11\pi}{4}+\sin\frac{15\pi}{4}\cos\frac{13\pi}{4}$

$\tan(3\pi-\frac{\pi}{4}) \to \underbrace{\tan(\pi-\frac{\pi}{4})}_{\text{ربع دوم}} = (-\tan\frac{\pi}{4}) = -1$

$\sin\frac{15\pi}{4} \to \underbrace{\sin(4\pi-\frac{\pi}{4})}_{\text{ربع چهارم}} = -\sin(2\pi-\frac{\pi}{4}) = -\frac{\sqrt{2}}{2}$

$\cos(3\pi+\frac{\pi}{4}) \to -\underbrace{\cos(\pi+\frac{\pi}{4})}_{\text{ربع سوم}} = \frac{-\sqrt{2}}{2}$

$-1-\left(\frac{\sqrt{2}}{2}\right)\left(-\frac{\sqrt{2}}{2}\right) = -1+\frac{1}{2} = -\frac{1}{2}$

ب) $\tan\frac{17\pi}{6}\sin\frac{11\pi}{3}+\cos\frac{10\pi}{3}$

$\tan(3\pi-\frac{\pi}{6}) = \tan(\pi-\frac{\pi}{6}) = -\tan\frac{\pi}{6} = \frac{-\sqrt{3}}{3}$

$\sin\frac{11\pi}{3} = \sin(4\pi-\frac{\pi}{3}) = \sin(2\pi-\frac{\pi}{3}) = \sin(-\frac{\pi}{3}) \to -\frac{\sqrt{3}}{2}$

$\cos\frac{10\pi}{3} = \cos(3\pi+\frac{\pi}{3}) = \cos(\pi+\frac{\pi}{3}) = -\cos\frac{\pi}{3} = -\frac{1}{2}$

$\left(\frac{-\sqrt{3}}{3}\right)\left(\frac{-\sqrt{3}}{2}\right)-\frac{1}{2} = \frac{1}{2}-\frac{1}{2} = 0$

۲- $\cot x = 4$ $\qquad \frac{3\cos x-\sin x}{\cos x+\sin x} = \frac{12\sin x-\sin x}{4\sin x+\sin x} = \frac{11\sin x}{5\sin x} = \boxed{\frac{11}{5}}$

$\frac{\cos x}{\sin x} = 4 \to 4\sin x = \cos x$

۳- $\sin\alpha = 2\cos\alpha$

ربع سوم

$\cos x = ?$

$\sin^2 x+\cos^2 x = 1$

$(2\cos x)^2+\cos^2 x = \underbrace{5\cos^2 x = 1}$

$\cos^2 x = \frac{1}{5} \to \cos x = \pm\frac{1}{\sqrt{5}}$

ربع سوم $\to \cos<0 \to -\frac{1}{\sqrt{5}}\times\frac{\sqrt{5}}{\sqrt{5}} = \boxed{\frac{-\sqrt{5}}{5}}$

۴- الف) $\sin x = \frac{\sqrt{2}}{10}$

$\cos(\frac{11\pi}{4}+x)$

$\boxed{2\pi+\frac{3\pi}{4}+x}$

$1 = \sin^2 x+\cos^2 x \to \sin^2 x = \frac{2}{100} = \frac{1}{50}$

$\cos^2 x \to \frac{49}{50}$

$\cos x = \pm\sqrt{\frac{49}{50}} = \pm\frac{7}{\sqrt{50}} \to \pm\frac{7}{5\sqrt{2}}$

$$\cos\left(\frac{3\pi}{4}+x\right)=\underbrace{\cos\frac{3\pi}{4}}_{-\frac{\sqrt{2}}{2}}\cos x-\underbrace{\boxed{\sin\frac{3\pi}{4}}}_{\frac{\sqrt{2}}{2}}\sin x$$

$$-\frac{\sqrt{2}}{2}(\cos x+\sin x)$$

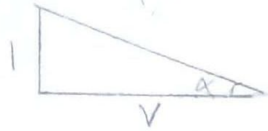
$$\cos x+\sin x=\frac{-7}{5\sqrt{2}}+\frac{\sqrt{2}}{10\sqrt{2}}=\frac{-6}{5\sqrt{2}}$$

$$\cos\left(\frac{11\pi}{4}+x\right)=-\frac{\sqrt{2}}{2}\times\left(\frac{-6}{5\sqrt{2}}\right)=\frac{-1}{2}\times\frac{-6}{5}=\frac{6}{10}$$

$$\frac{3}{5}$$

ب)

$$\tan=\frac{1}{7}$$



$$\tan=\frac{\text{مقابل}}{\text{مجاور}}=\frac{1}{7}\qquad \text{وتر}=\sqrt{1+49}\rightarrow\sqrt{50}=5\sqrt{2}$$

$$\sin\left(\frac{13\pi}{4}+\alpha\right)=\underbrace{\sin\alpha+\cos\alpha}\qquad \begin{cases}\boxed{\sin\alpha=\frac{1}{5\sqrt{2}}}\\ \boxed{\cos\alpha=\frac{7}{5\sqrt{2}}}\end{cases}$$

$$\boxed{\frac{8}{5\sqrt{2}}}$$

$$\sin\left(3\pi+\frac{\pi}{4}+\alpha\right)=\sin\left(\pi+\frac{\pi}{4}+\alpha+\cancel{2\pi}\right)$$

$$\sin(\pi+\theta)=-\sin\theta\leftarrow\sin\left(\pi+\underbrace{\left(\frac{\pi}{4}+\alpha\right)}_{\theta}\right)$$

$$\alpha+\frac{\pi}{4}=\theta\rightarrow-\sin\left(\frac{\pi}{4}+\alpha\right)$$

$$\sin\left(\frac{\pi}{4}+\alpha\right)=\sin\frac{\pi}{4}\cos\alpha+\cos\frac{\pi}{4}\sin\alpha$$

$$\boxed{\sin\frac{\pi}{4}=\cos\frac{\pi}{4}=\frac{\sqrt{2}}{2}}\qquad \frac{\sqrt{2}}{2}\cos\alpha+\frac{\sqrt{2}}{2}\sin\alpha=$$

$$\frac{\sqrt{2}}{2}(\sin\alpha+\cos\alpha)$$

$$\frac{\sqrt{2}}{2}\times\frac{8}{5\sqrt{2}}=\frac{8}{2\times5}=\frac{4}{5}$$

$$\sin\left(\frac{13\pi}{4}+\alpha\right)=\boxed{-\frac{4}{5}}$$

۵ -

$$2\sin^2x+\cos^2x=\frac{4}{5}$$

$$2(1-\cos^2x)+\cos^2x=\frac{4}{3}$$

$$2-2\cos^2x+\cos^2x=\frac{4}{3}$$

$$2-\cos^2x=\frac{4}{3}\rightarrow\boxed{\cos^2x=\frac{2}{3}}$$

$$\sin^2x=1-\cos^2x=1-\frac{2}{3}=\boxed{\frac{1}{3}}$$

$$\tan^2x=\frac{\frac{1}{3}}{\frac{2}{3}}=\boxed{\frac{1}{2}}$$

-۶

$۴,۵ \rightarrow \frac{۹}{۲} = \frac{۱}{۲} \times ۶ \times \sqrt{۳}\ \sin\alpha$

$\sin\alpha = \frac{۳}{۲\sqrt{۳}} \times \frac{\sqrt{۳}}{\sqrt{۳}} = \frac{۳\sqrt{۳}}{۶} = \left(\frac{\sqrt{۳}}{۲}\right)$

$\alpha_{min} = ۶۰$

$\alpha_{max} = ۱۲۰ \quad \times ۲$

$\sin +$

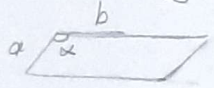
ربع اول $\rightarrow ۶۰° \rightarrow \frac{\sqrt{۳}}{۲}$

۲ برابر

ربع دوم $\rightarrow ۱۲۰° \rightarrow \frac{\sqrt{۳}}{۲}$

-۷

$S = ۴۵$



$\frac{a}{b} = \frac{۲}{۳}$

$S = a \cdot b \cdot \sin\alpha$

بنابراین $\rightarrow (۲K) \cdot (۳K) \cdot \sin ۱۵۰°$

$\sin ۳۰ = \frac{۱}{۲}$

$S = ۶K^۲ \times \frac{۱}{۲} = ۳K^۲$

$۳K^۲ = ۴۵ \rightarrow K^۲ = ۱۸ \rightarrow K = ۳\sqrt{۲}$

اضلاع $=$ $a = ۲K = ۶\sqrt{۲}$

$b = ۳K = ۹\sqrt{۲}$

$P_{محیط} = ۲a + ۲b = ۲(۶\sqrt{۲} + ۹\sqrt{۲}) = \boxed{۳۰\sqrt{۲}}$

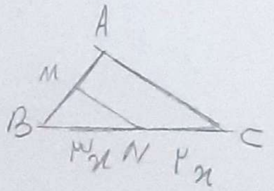
-۸

$\sin A \left(\frac{۱}{۲} \times ۵ \times ۷ - \frac{۱}{۲} \times ۴ \times ۷\right) = \frac{۷}{۴}$

$\frac{۷}{۲} \sin A = \frac{۷}{۴} \rightarrow \sin A = \frac{۲}{۴} = \frac{۱}{۲}$

$A = ۳۰° \rightarrow \tan A = \frac{\sqrt{۳}}{۳}$

-۹



$\frac{S_{BNM}}{S_{ABC}} = \frac{BM \times BN}{BA \times BC} = \frac{BM}{AB} \times \frac{۳}{۵} = \frac{۱}{۳}$

$\frac{BM}{AB} = \frac{۵}{۹} \rightarrow \frac{BM}{AM} = \frac{۵}{۴} = ۱,۲۵$

-۱۰

$-\frac{۱}{\cot\alpha} = -\tan\alpha \rightarrow -\tan\alpha = \frac{۱+\sin\alpha}{\cos\alpha}$ ربع ۲

ربع ۲،۳ $\leftarrow \ominus \cos\alpha$

$+ \frac{۱+\sin\alpha}{\cos\alpha}$ +

$-۱ \leq \sin\alpha \leq ۱$

$\frac{۱}{\sqrt{\cos^۲\alpha}} \ominus (\tan\alpha)$ ربع دوم چهارم

$+ \rightarrow$

ربع ۲