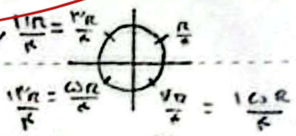
نام: فاطمه احمدی　　۲۰　　شماره تکلیف: ۱۸　　کلاس: دهم دختر A

۱- الف) $\tan\frac{11\pi}{4} + \sin\frac{15\pi}{4}\cos\frac{13\pi}{4} \rightsquigarrow 2\pi = \frac{8\pi}{4}$ ← دور کامل



$\tan\frac{11\pi}{4} = \tan\frac{3\pi}{4} \rightsquigarrow \frac{\frac{\sqrt2}{2}}{-\frac{\sqrt2}{2}} = -1$ / $\sin\frac{15\pi}{4} = \frac{7\pi}{4} = \frac{-\sqrt2}{2}$ / $\cos\frac{13\pi}{4} = \cos\frac{5\pi}{4} = -\frac{\sqrt2}{2}$

$-1 + (\underbrace{-\frac{\sqrt2}{2} \times -\frac{\sqrt2}{2}}_{\frac{2}{4}=\frac12}) = -1 + \frac12 = \frac{-1}{2}$

۲

ب) $\tan\frac{17\pi}{6}\sin\frac{11\pi}{3} + \cos\frac{10\pi}{3} \rightsquigarrow 2\pi = \frac{12\pi}{6} \to \frac{17\pi}{6} = \frac{\overset{2\pi}{12\pi}}{6} + \boxed{\frac{5\pi}{6}} \rightsquigarrow \tan\frac{17\pi}{6} = \tan\frac{5\pi}{6} = \frac{\frac12}{-\frac{\sqrt3}{2}}$ $(-\frac{\sqrt3}{3})$

$2\pi = \frac{6\pi}{3} \rightsquigarrow \frac{\overset{2\pi}{6\pi}}{3} + \frac{5\pi}{3} \rightsquigarrow \sin\frac{11\pi}{3} = \frac{5\pi}{3} = -\frac{\sqrt3}{2}$, $\frac{10\pi}{3} = \frac{\overset{2\pi}{6\pi}}{3} + \frac{4\pi}{3} \rightsquigarrow \cos\frac{10\pi}{3} = \cos\frac{4\pi}{3} = \frac{-1}{2}$

$(\underbrace{-\frac{\sqrt3}{3} \times -\frac{\sqrt3}{2}}_{\frac{3}{6}=\frac12}) - \frac12 = \frac12 - \frac12 = 0$

۲- $\cot x = 4 \rightsquigarrow 1+\cot^2 x = \frac{1}{\sin^2 x} \rightsquigarrow \underset{17}{1+16} = \frac{1}{\sin^2 x} \rightsquigarrow 17\sin^2 x = 1 \rightsquigarrow \sin x = \pm\frac{1}{\sqrt{17}} = \frac{\sqrt{17}}{17}$

$\sin^2 x = \frac{1}{17}$

$\sin^2 x + \cos^2 x = 1 \rightsquigarrow \cos^2 x = \frac{17}{17} - \frac{1}{17} = \frac{16}{17} \rightsquigarrow \cos x = \pm\frac{4}{\sqrt{17}} = \pm\frac{4\sqrt{17}}{17}$ ⇝ sin و cos هم علامت چون cot > ۰

$\frac{3\cos x - \sin x}{\cos x + \sin x} \rightsquigarrow \frac{12\frac{\sqrt{17}}{17} - \frac{\sqrt{17}}{17}}{\frac{4\sqrt{17}}{17} + \frac{\sqrt{17}}{17}} = \frac{\frac{11\sqrt{17}}{17}}{\frac{5\sqrt{17}}{17}} = \frac{11}{5} = 2/2$

۵

اگر sin x و cos x را منفی در نظر بگیریم نیز حاصل $\frac{11}{5}$ می‌شود پس جواب تفاوتی نمی‌کند

۳- $\sin\alpha = 2\cos\alpha \rightsquigarrow \frac{\sin\alpha}{\cos\alpha} = \tan\alpha = 2$, $\alpha \to$ ربع ۳ , $\cos\alpha = ?$

ع . ن ن چون ربع ۳ است

$1+\tan^2\alpha = \frac{1}{\cos^2\alpha} \rightsquigarrow 1 + \underset{2^2}{4} = \frac{1}{\cos^2\alpha} \rightsquigarrow 5 = \frac{1}{\cos^2\alpha} \rightsquigarrow \cos^2\alpha = \frac15 \rightsquigarrow \cos\alpha = \pm\frac{1}{\sqrt5}$

پس $\cos\alpha = \frac{-1}{\sqrt5} = -\frac{\sqrt5}{5}$

۳

۴- الف) α ربع ۲ / $\sin\alpha = \frac{\sqrt2}{10} \rightsquigarrow \cos(\frac{11\pi}{4} + \alpha)$

۹

ح . ن ن چون در ربع ۲

$\sin^2\alpha + \cos^2\alpha = 1 \rightsquigarrow \cos^2\alpha = 1 - \sin^2\alpha \rightsquigarrow \cos^2\alpha = 1 - \frac{2}{100} = \frac{98}{100} = \frac{49}{50} \to \cos\alpha = \pm\frac{7}{\underset{5\sqrt2}{\sqrt{50}}}$

$\cos\alpha = \frac{-7}{5\sqrt2} = \frac{-7\sqrt2}{10}$, $\cos\frac{11\pi}{4} \overset{*}{=} -\frac{\sqrt2}{2} \rightsquigarrow \cos(\frac{11\pi}{4} + \alpha) = (-\frac{7\sqrt2}{10})(-\frac{\sqrt2}{2}) - (\frac{\sqrt2}{10})(\frac{\sqrt2}{2}) = \frac{14}{20} - \frac{2}{20} = \frac{12}{20}$

$* \; 2\pi = \frac{8\pi}{4} \to \frac{8\pi + 3\pi}{4} \rightsquigarrow \frac{11\pi}{4} = \frac{3\pi}{4} \to \cos\frac{11\pi}{4} = \cos\frac{3\pi}{4}$

«$\frac{12}{20} = \frac35$»

ادامه سؤال ۴ -

ب) α ربع ۳ / $\tan\alpha = \frac{۱}{۷}$ $\sin\left(\frac{۱۳\pi}{۴}+\alpha\right)$

$*\sin(\alpha+\beta) = \sin\alpha\cos\beta + \sin\beta\cos\alpha$

$1+\tan^۲\alpha = \frac{1}{\cos^۲\alpha} \rightsquigarrow 1 + \frac{1}{۴۹} = \frac{1}{\cos^۲\alpha} \Rightarrow \frac{۴۹}{۵۰} = \cos^۲\alpha \to \cos\alpha = \pm\frac{۷}{۵\sqrt{۲}} = +\frac{۷\sqrt{۲}}{۱۰}$

ربع ۳ ، غ.ق.ق

$\sin^۲\alpha = 1-\cos^۲\alpha = 1 - \frac{۴۹}{۵۰} = \frac{1}{۵۰} = \sin^۲\alpha \to \sin\alpha = \pm\frac{1}{۵\sqrt{۲}} = +\frac{\sqrt{۲}}{۱۰}$

غ.ق.ق

$۳\pi = \frac{۸\pi}{۴} + \frac{۵\pi}{۴} \to \sin\frac{۵\pi}{۴} = \sin\frac{۱۳\pi}{۴} = \frac{-\sqrt{۲}}{۲} \rightsquigarrow \sin\left(\frac{۱۳\pi}{۴}+\alpha\right) = \left(-\frac{\sqrt{۲}}{۲}\right)\left(\frac{۷\sqrt{۲}}{۱۰}\right) + \left(\frac{\sqrt{۲}}{۱۰}\right)\left(-\frac{\sqrt{۲}}{۲}\right) =$

$\frac{-۱۴}{۲۰} + \left(\frac{-۲}{۲۰}\right) = \frac{-۱۶}{۲۰} = \underline{\underline{\frac{-۴}{۵}}}$

۵- (۲)

$۲\sin^۲x + \cos^۲x = \frac{۴}{۳}$ ، $\tan^۲x = ?$

$\sin^۲x = 1-\cos^۲x \rightsquigarrow ۲(1-\cos^۲x)+\cos^۲x = \frac{۴}{۳} \rightsquigarrow -\cos^۲x + \underset{\frac{۶}{۳}}{۲} = \frac{۴}{۳} \rightsquigarrow \cancel{-}\cos^۲x = \frac{۴-۶}{۳} = \cancel{-}\frac{۲}{۳}$

$\cos^۲x = \frac{۲}{۳}$ ، $\sin^۲x = 1-\cos^۲x = 1-\frac{۲}{۳} = \frac{1}{۳}$ $\tan^۲x = \frac{\sin^۲x}{\cos^۲x} = \frac{\frac{1}{۳}}{\frac{۲}{۳}} = \boxed{\frac{1}{۲}}$

۶- (۲)

$\sqrt{۳}$ ، ۶ ، $\alpha \to$ زاویه بین $S = ۴٫۵$ $\frac{\max\alpha}{\min\alpha} = ?$

$S = \frac{1}{۲}ab\sin\alpha \to \frac{1}{\cancel{۲}} \times \sqrt{۳} \times \overset{۳}{\cancel{۶}} \times \sin\alpha = ۴٫۵ \rightsquigarrow ۳\sqrt{۳} \times \sin\alpha = ۴٫۵$

$\sin\alpha = \frac{\frac{\overset{۱۵}{\cancel{۴۵}}}{۱۰}}{\frac{۳\sqrt{۳}}{۱}} = \frac{۱۵^{۳}}{۱۰\sqrt{۳}_{۲}} = \frac{۳}{۲\sqrt{۳}} \times \frac{\sqrt{۳}}{\sqrt{۳}} = \frac{\cancel{۳}\sqrt{۳}}{\cancel{۶}} = \frac{\sqrt{۳}}{۲} \rightsquigarrow \alpha = ۶۰^\circ$ یا ۱۲۰° $\frac{۱۲۰}{۶۰} = \boxed{۲}$

۷- (۲)

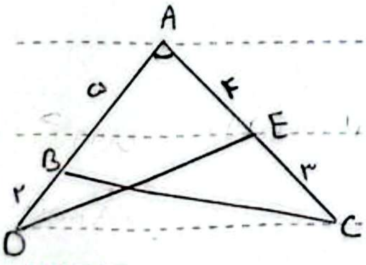
$\frac{a}{b} = \frac{۲}{۳}$ $S = ab\sin\alpha = ۵۴$ (۱۵۰° ـا) $\rightsquigarrow (۲k)(۳k)\left(\frac{1}{۲}\right) = ۵۴ \to ۶k^۲ = ۵۴\times۲ = ۱۰۸$

ـا $a = ۲k$ ، $b = ۳k$ $\sin ۱۵۰^\circ = \frac{1}{۲}$ ـا $k^۲ = ۱۸ \to k = \pm\sqrt{۱۸} = ۳\sqrt{۲}$

غ.ق.ق

$a = ۲(۳\sqrt{۲}) = ۶\sqrt{۲}$ ، $b = ۳(۳\sqrt{۲}) = ۹\sqrt{۲}$ → محیط متوازی الاضلاع $= ۲(a+b) = ۲(\underbrace{۶\sqrt{۲}+۹\sqrt{۲}}_{۱۵\sqrt{۲}}) = \underset{\sim}{۳۰\sqrt{۲}}$

۸- (۲)



$S_{ABC} - S_{ADE} = ۱٫۷۵$

$\left(\frac{1}{۲}\times۵\times۷\sin A\right) - \left(\frac{1}{۲}\times۴\times۷\times\sin\alpha\right) = ۱٫۷۵ \to \frac{۳۵}{۲}\sin\alpha - ۱۴\sin\alpha = ۳٫۵\alpha = ۱٫۷۵$

$\sin\alpha = \frac{۱٫۷۵}{۳٫۵} = \frac{۵}{۱۰} = \frac{1}{۲}$ $\xrightarrow{\text{زاویه حاده}}$ $\cos\alpha = \frac{\sqrt{۳}}{۲} \rightsquigarrow \underset{\sim}{\alpha = ۳۰^\circ}$

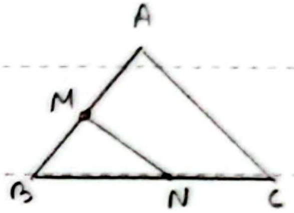
$\tan\alpha = \frac{\frac{1}{۲}}{\frac{\sqrt{۳}}{۲}} = \boxed{\frac{\sqrt{۳}}{۳}}$

Subject:

Date : / /

$\overline{NC} = \frac{۲}{۳}\overline{BN}$

$۲\overline{BN} = ۳\overline{NC}$, $S_{ABC} = ۳ S_{BMN}$, $\frac{BM}{AM} = ?$ -9



$S_{ABC} = \cancel{\frac{۱}{۲}} \times \overline{AB} \times \overline{BC} \times \sin B = ۳ \times \underbrace{\left(\frac{۱}{۲} \times \overline{BM} \times \overline{BN} \times \cancel{\sin B}\right)}_{S_{BMN}}$

$(\overline{AB} \times \overline{BC}) = ۳ \times \overline{BM} \times \overline{BN} \Rightarrow (\overline{AM} + \overline{BM})\underbrace{(\overline{BN} + \overline{NC})}_{\frac{۵}{۳}\overline{BN}} = ۳(\overline{BM} \times \overline{BN})$

$\frac{۵}{۳}\overline{BN}(\overline{AM} + \overline{BM}) = ۳ \times \overline{BM} \times \overline{BN} \rightsquigarrow \underbrace{\frac{۵}{۳}(\overline{AM} + \overline{BM})}_{\frac{۵}{۳}\overline{AM} + \frac{۵}{۳}\overline{BM}} = ۳\overline{BM} \rightsquigarrow (\frac{۵}{۳}\overline{AM} = \frac{۴}{۳}\overline{BM}) ۳$

$۵\overline{AM} = ۴\overline{BM} \rightarrow \overline{BM} = \frac{۵}{۴}\overline{AM} \rightsquigarrow$ « $\frac{BM}{AM} = \frac{۵}{۴}$ »

$\frac{۱}{\sqrt{\cos^۲\alpha}} - \tan\alpha = \frac{۱ + \sin\alpha}{|\cos\alpha|}$, $\underbrace{\frac{\overset{+}{\widetilde{|\sin\alpha|}}}{\cos\alpha}}$ $= \left(-\frac{۱}{\cot\alpha}\right) \Rightarrow |\sin\alpha| = -\sin\alpha \rightarrow \underset{\sim}{\sin\alpha < 0}$ ① -10

$\hookrightarrow \frac{۱}{|\cos\alpha|} - \frac{\sin\alpha}{\cos\alpha} = \frac{۱ - \sin\alpha}{\cos\alpha}$ (if $\cos > 0$) } $\hookrightarrow \frac{-\sin\alpha}{\cos\alpha}$

$\hookrightarrow \frac{۱ - \sin\alpha}{\cos\alpha} \rightsquigarrow \frac{۱ + \sin\alpha}{\cos\alpha} \rightsquigarrow ۱ - \sin\alpha = ۱ + \sin\alpha \rightarrow \sin\alpha = 0 \longrightarrow \sin < 0$ نیست ✗

(if $\cos < 0$) $\Rightarrow \frac{۱}{-\cos\alpha} + \frac{\sin\alpha}{-\cos\alpha} = \frac{۱ + \sin\alpha}{-\cos\alpha} \rightsquigarrow \frac{۱ + \sin\alpha}{-\cos\alpha} = \frac{۱ + \sin\alpha}{|\cos\alpha|} \Rightarrow \underset{\sim}{\cos\alpha < 0}$ ② ✓

$\sin < 0$, $\cos < 0 \rightsquigarrow$ (ربع ۳)