پرنیان باقری/ پاسخنامه تشریحی تکلیف شماره ۱۸/ کلاس دهم دقیق A ۲۵

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الف) $\tan\frac{11\pi}{4} + \sin\frac{15\pi}{4}\cos\frac{13\pi}{4} = \tan\frac{3\pi}{4} + \sin\frac{7\pi}{4}\cos\frac{5\pi}{4} = \tan_{135} + \sin_{315}\cos_{225} = -1 + \frac{-\sqrt{2}}{2}\times\frac{-\sqrt{2}}{2} = \frac{-1}{2}$

ب) $\tan\frac{17\pi}{6}\sin\frac{11\pi}{3} + \cos\frac{10\pi}{3} = \tan\frac{5\pi}{6}\sin\frac{5\pi}{3} + \cos\frac{4\pi}{3} = \tan_{150}\sin_{300} + \cos_{240} = \frac{-\sqrt{3}}{3}\times\frac{-\sqrt{3}}{2} + \frac{-1}{2} = 0$

۲

$\cot_x = 4 \rightsquigarrow \frac{\cos_x}{\sin_x} = 4 \rightsquigarrow \cos_x = 4\sin_x$

$\frac{3\cos_x - \sin_x}{\cos_x + \sin_x} = \frac{12\sin x - \sin x}{4\sin_x + \sin x} = \frac{11\sin_x}{5\sin_x} = \frac{11}{5}$

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$\sin_a = 2\cos_a \ , \ 180 < a < 270$

$\sin^2_a + \cos^2_a = 1 \rightsquigarrow 4\cos^2_a + \cos^2_a = 5\cos^2_a = 1 \rightsquigarrow \cos^2_a = \frac{1}{5} \rightsquigarrow \cos_a = \oplus\frac{1}{\sqrt{5}}$

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الف) $\cos(\frac{11\pi}{4} + a) = \cos(\frac{3\pi}{4} + a) = \cos_{\frac{3\pi}{4}}\times\cos_a - \sin_{\frac{3\pi}{4}}\times\sin_a = (-\frac{\sqrt{2}}{2}\times\frac{-7\sqrt{2}}{10}) - (\frac{\sqrt{2}}{2}\times\frac{\sqrt{2}}{10}) = \frac{12}{20} = \frac{3}{5}$

$[\cos_a = \sqrt{1 - \frac{2}{100}} = \oplus\frac{7\sqrt{2}}{10}]$

ب) $\sin(\frac{13\pi}{4} + a) = \sin(\frac{5\pi}{4} + a) = \sin_{\frac{5\pi}{4}}\cos_a + \sin_a\cos_{\frac{5\pi}{4}} = (-\frac{\sqrt{2}}{2}\times\frac{7\sqrt{2}}{10}) + (\frac{-\sqrt{2}}{2}\times\frac{\sqrt{2}}{10}) = \frac{-16}{20} = \frac{-4}{5}$

$[1 + \tan^2_a = \frac{1}{\cos^2_a} \rightsquigarrow \frac{49}{50} = \cos^2_a \rightsquigarrow \cos_a = \frac{7}{5\sqrt{2}} \ , \ \sin_a = \tan_a\times\cos_a = \frac{1}{7}\times\frac{7}{5\sqrt{2}} = \frac{1}{5\sqrt{2}} = \frac{\sqrt{2}}{10}]$

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$\begin{cases} 2\sin^2_x + \cos^2_x = \frac{4}{3} \\ \sin^2_x + \cos^2_x = 1 \end{cases} \Rightarrow 2\sin^2_x + \cos^2_x - \sin^2_x - \cos^2_x = \sin^2_x = \frac{1}{3} \rightsquigarrow \cos^2_x = \frac{2}{3}$

$\tan^2_x + 1 = \frac{1}{\cos^2_x} \rightsquigarrow \tan^2_x = \frac{3}{2} - 1 = \frac{1}{2}$

۶

$\frac{1}{2}\times a\times b\times\sin_c = \frac{1}{2}\times 6\times\sqrt{3}\times\sin_a = 4.5 \rightsquigarrow \sin_a = \frac{3}{2\sqrt{3}} = \frac{\sqrt{3}}{2} \rightsquigarrow a = 60°$ یا $120°$

$\frac{120}{60} = 2$

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$a\times b\times\sin_{150} = 54 \rightsquigarrow \frac{3}{2}a^2\times\frac{1}{2} = 54 \rightsquigarrow a^2 = 72 \rightsquigarrow a = 6\sqrt{2} \ , \ b = 9\sqrt{2}$

$[\frac{a}{b} = \frac{2}{3} \rightsquigarrow b = \frac{3}{2}a]$

$P_{\text{متوازی الاضلاع}} = 2a + 2b = 12\sqrt{2} + 18\sqrt{2} = 30\sqrt{2}$

۸

$S_{A\hat{B}C} = 5\times 7\times\sin_A\times\frac{1}{2} = \frac{35}{2}\sin_A$

$S_{A\hat{D}E} = 4\times 7\times\sin_A\times\frac{1}{2} = 14\sin_A$

$S_{A\hat{B}C} - S_{A\hat{D}E} = 3.5\sin_A = 1.75 \rightsquigarrow \sin_A = \frac{1}{2}$

$A = 30° \rightarrow \tan_{30} = \frac{\sqrt{3}}{3}$

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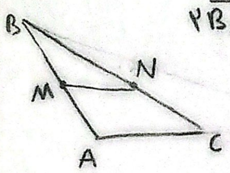

$2\overline{BN} = 3\overline{NC}$

$S_{A\hat{B}C} = 3S_{B\hat{M}N} \rightsquigarrow \frac{1}{2}\times\overline{AB}\times\overline{BC}\times\sin_B = 3\times\frac{1}{2}\times\overline{BM}\times\overline{BN}\times\sin_B$

$\overline{AB}\times\overline{BC} = 3\times\overline{BM}\times\overline{BN}$

$(\overline{BM} + \overline{AM})(\overline{BN} + \overline{NC}) = 3\overline{BN}\times\overline{BM}$ ($\overline{BN}+\overline{NC} = \frac{5}{3}\overline{BN}$) $\rightsquigarrow 3\overline{BM} = \frac{5}{3}(\overline{BM} + \overline{AM}) \rightsquigarrow 4\overline{BM} = 5\overline{AM} \rightsquigarrow \frac{\overline{BM}}{\overline{AM}} = \frac{5}{4}$

$$\frac{1}{\sqrt{\cos^2 a}} - \tan a = \frac{1+\sin a}{|\cos a|} \quad , \quad \frac{|\sin a|}{\cos a} = -\frac{1}{\cot a}$$

$$\downarrow$$

$$\frac{1}{|\cos a|} - \frac{\sin a}{\cos a} \xrightarrow{\cos a < 0} = \frac{1+\sin a}{|\cos a|}$$

$$\searrow$$

$$\frac{|\sin a|}{\cos a} = \frac{-\sin a}{\cos a} \rightsquigarrow \boxed{\sin a < 0}$$

انتهای کمان a در ناحیه سوم است.

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