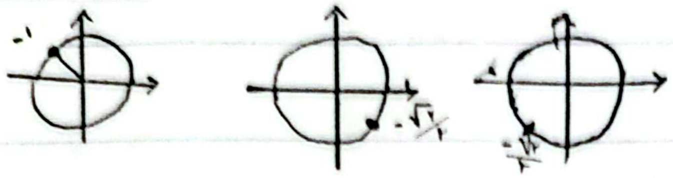
سوال ①

الف) $\tan\frac{11\pi}{4} + \sin\frac{15\pi}{4}\cos\frac{13\pi}{4}$

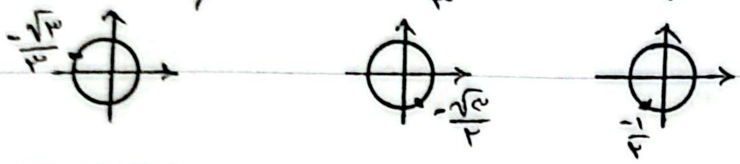
$\tan(2\pi+\frac{3\pi}{4}) + \sin(4\pi-\frac{\pi}{4})\cos(2\pi+\frac{5\pi}{4}) = (-1) + (\frac{-\sqrt{2}}{2})(\frac{-\sqrt{2}}{2}) = -1+\frac{1}{2} = \boxed{-\frac{1}{2}}$



ب) $\tan\frac{17\pi}{6}\sin\frac{11\pi}{3} + \cos\frac{10\pi}{3}$

$\tan(3\pi-\frac{\pi}{6})\sin(3\pi+\frac{2\pi}{3}) + \cos(3\pi+\frac{\pi}{3}) = (-\frac{\sqrt{3}}{3})(-\frac{\sqrt{3}}{2}) + (-\frac{1}{2})$

$= \frac{3}{6} - \frac{3}{6} = \boxed{0}$



سوال ②

$\cot x = 4 \rightarrow \frac{3\cos x - \sin x}{\cos x + \sin x} = ?$

$$\frac{\frac{3\cos x}{\sin x} - \frac{\sin x}{\sin x}}{\frac{\cos x}{\sin x} + \frac{\sin x}{\sin x}} = \frac{12-1}{4+1} = \boxed{\frac{11}{5}}$$

سوال ③

$\sin\alpha = 2\cos\alpha$ ، ربع سوم $\Rightarrow \cos\alpha = ?$

$\sin^2\alpha + \cos^2\alpha = 1 \Rightarrow (2\cos\alpha)^2 + \cos^2\alpha = 1 \Rightarrow 4\cos^2\alpha + \cos^2\alpha = 1$

$\Rightarrow \cos^2\alpha = \frac{1}{5} \Rightarrow \cos\alpha = \ominus\frac{\sqrt{5}}{5}$ (منفی) $\Rightarrow 5\cos^2\alpha = 1$

سوال ④

$\cos(4\pi + \frac{5\pi}{4}) = (\frac{5\pi}{4} + \alpha)$

الف) ربع ۲ و $\sin\alpha = \frac{\sqrt{2}}{10} \Rightarrow \cos(\frac{21\pi}{4} + \alpha) = ?$

$\sin\alpha > 0$
$\cos\alpha < 0$

$\sin^2\alpha + \cos^2\alpha = 1 \Rightarrow (\frac{\sqrt{2}}{10})^2 + \cos^2\alpha = 1$ $\quad \frac{2}{100} = \frac{1}{50}$

$\Rightarrow \cos^2\alpha = \frac{50}{50} - \frac{1}{50} = \frac{49}{50} \Rightarrow \cos\alpha = \frac{7\sqrt{2}}{10}$

$\cos(\frac{3\pi}{4} + \alpha) = \underbrace{\cos\frac{3\pi}{4}}_{-\frac{\sqrt{2}}{2}}\underbrace{\cos\alpha}_{-\frac{7\sqrt{2}}{10}} - \underbrace{\sin\frac{3\pi}{4}}_{\frac{\sqrt{2}}{2}}\underbrace{\sin\alpha}_{\frac{\sqrt{2}}{10}} = \frac{7}{10} - \frac{1}{10} = \frac{6}{10} = \boxed{\frac{3}{5}}$

ب) داریم $\tan\alpha = \frac{1}{7} \Rightarrow \sin\left(\frac{11\pi}{4} + \alpha\right) = ?$

$1 + \tan^2\alpha = \frac{1}{\cos^2\alpha} \Rightarrow 1 + \left(\frac{1}{7}\right)^2 = \frac{1}{\cos^2\alpha} \Rightarrow \frac{1}{\cos\alpha} = \frac{5\sqrt{2}}{7}$

$\sin\left(\frac{5\pi}{4} + \alpha\right) = \sin\frac{5\pi}{4}\cos\alpha + \cos\frac{5\pi}{4}\sin\alpha = -\frac{\sqrt{2}}{2}\cos\alpha - \frac{\sqrt{2}}{2}\sin\alpha$

$= -\frac{\sqrt{2}}{2}(\cos\alpha + \sin\alpha)$

① $\cos\alpha = \frac{7\sqrt{2}}{10}$

② $\sin\alpha = \tan\alpha \cdot \cos\alpha = \frac{1}{7} \times \frac{7\sqrt{2}}{10} = \frac{\sqrt{2}}{10}$

$\Rightarrow -\frac{\sqrt{2}}{2}\left(\frac{7\sqrt{2}}{10} + \frac{\sqrt{2}}{10}\right) = -\frac{\sqrt{2}}{2} \times \frac{4\sqrt{2}}{5} = -\frac{4}{5}$

سوال ۵

$\tan x = ?$ $\quad 2\sin^2 x + \cos^2 x = \frac{4}{3}$

$\sin^2 x + \sin^2 x + \cos^2 x = \frac{4}{3} \Rightarrow \sin^2 x + 1 = \frac{4}{3}$

$\sin^2 x = \frac{4}{3} - \frac{3}{3} = \frac{1}{3}$

$\frac{1}{5} + \cos 2x = 1 \Rightarrow \cos^2 x = \frac{3}{5} - \frac{1}{5} = \frac{2}{5}$

$\tan^2 x = \frac{\sin^2 x}{\cos^2 x} = \frac{\frac{1}{3}}{\frac{2}{5}} = \frac{1}{2}$

سوال ۶

$\sqrt{3}, 4, \alpha \quad S = 4\sqrt{5} \quad \frac{\max\alpha}{\min\alpha}$

$4\sqrt{5} = \frac{1}{2}ab\sin\alpha \Rightarrow 4\sqrt{5} = \frac{1}{2} \times \sqrt{3} \times 4 \times \sin\alpha \Rightarrow 4\sqrt{5} = 2\sqrt{3}\sin\alpha$

$\sin\alpha = \frac{4\sqrt{5}}{2\sqrt{3}} = \frac{\sqrt{15}}{2} \Rightarrow 0 < \alpha < 60^\circ \text{ یا } 120^\circ < \alpha < \pi$

$\frac{\pi}{3} \to 60^\circ$ (min) $\qquad \frac{2\pi}{3} \to 120^\circ$ (max)

$\frac{120}{60} = 2 \Rightarrow \frac{\frac{2\pi}{3}}{\frac{\pi}{3}} = 2$

تورج عباسی

سوال ۷

$S = 54 \qquad \frac{a}{b} = \frac{2}{3}$ زاویه بین دو ضلع ۱۵۰°

$54 = ab\sin\alpha \Rightarrow 54 = ab(\sin 150)$

$\sin(180-30) = \sin(30) = \frac{1}{2}$

$54 = ab\frac{1}{2} \Rightarrow ab = 54 \times 2 \Rightarrow ab = 108$

$a = \frac{2}{3}b \Rightarrow (\frac{2}{3}b)(b) = 108 \Rightarrow \frac{2}{3}b^2 = 108 \Rightarrow b^2 = 108 \times \frac{3}{2} \Rightarrow b^2 = 54 \times 3 \Rightarrow b^2 = 162$

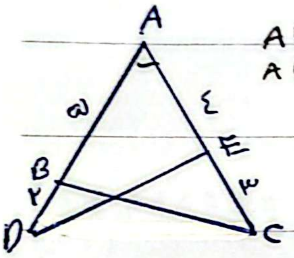
$b = 9\sqrt{2}$

$a = \frac{2}{3}(9\sqrt{2}) \Rightarrow a = 6\sqrt{2}$

$P = 2(a+b) = 2(6\sqrt{2}+9\sqrt{2}) = \boxed{30\sqrt{2}}$

سوال ۸:

$S_{ABC} - S_{ADE} = 1.75 \qquad \tan\hat{A} = ?$ (حاده)



$AD = 7$
$AC = 7$

$S_{(A\hat{B}C)} - S(A\hat{D}E) = 1.75 \qquad 0 < A < 90°$

$S = \frac{1}{2}ab\sin\theta$

$S_{(A\hat{B}C)} = \frac{1}{2}(AB)(AC)\sin A = \frac{1}{2} \times 5 \times 7 \times \sin A = \frac{35}{2}\sin A = 17.5\sin A$

$S(A\hat{D}E) = \frac{1}{2}(AD)(AE)\sin A = \frac{1}{2} \times 7 \times 4 \times \sin A = \frac{28}{2}\sin A = 14\sin A$

$(17.5\sin A) - (14\sin A) = 1.75 \Rightarrow 3.5\sin A = 1.75$

$\sin A = \frac{1.75}{3.5} = \frac{1}{2}$

$\sin^2 A + \cos^2 A = 1 \Rightarrow (\frac{1}{2})^2 + \cos^2 A = 1 \Rightarrow \frac{1}{4} + \cos^2 A = 1 \Rightarrow \cos^2 A = \frac{3}{4}$

$\cos A = \frac{\sqrt{3}}{2}$

$\tan A = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}} = \boxed{\frac{\sqrt{3}}{3}}$

Subject: Date: Sa Su Mo Tu We Th Fr

AB∋M , BC∋N $\quad 2\overline{BN} = 3\overline{NC} \quad S(\widehat{ABC}) = 3S(\widehat{BMN})$ سوال ۹

$\Rightarrow \frac{BN}{NC} = \frac{3}{2}$

B مشترک

$\Rightarrow BC = BN + NC = 3K + 2K = 5K$

$\frac{BM}{BC} = \frac{3K}{5K} = \frac{3}{5}$

$\frac{S_{BMN}}{S_{ABC}} = \frac{BM \times BN}{BA \times BC} = \frac{1}{3} \xRightarrow{\frac{BN}{BC} = \frac{3}{5}} \frac{1}{3} = \frac{BM}{BA} \times \frac{3}{5}$

$\frac{BM}{BA} = \frac{1}{3} \times \frac{5}{3} = \frac{5}{9}$

$BA = BM + AM$

$\frac{BM}{BM + AM} = \frac{5}{9} \longrightarrow \frac{BM}{AM} = \frac{5}{9-5} = \frac{5}{4}$

سوال ۱۰ :

$\frac{1}{\sqrt{\cos^2\alpha}} - \tan\alpha = \frac{1+\sin\alpha}{|\cos\alpha|} \Rightarrow \cos\alpha \neq 0 \Rightarrow \cot\alpha \neq 0$

$\Rightarrow \sin\alpha \neq 0$

$\frac{|\sin\alpha|}{\cos\alpha} = -\frac{1}{\cot\alpha}$

$-\frac{\sin\alpha}{\cos\alpha} \Rightarrow |\sin\alpha| = -\sin\alpha$

$\sin\alpha \leq 0$, $\sin\alpha \neq 0$

$-\frac{\sin\alpha}{\cos\alpha} = -\frac{\sin\alpha}{\cos\alpha} \rightarrow \sin\alpha < 0 \quad \cos\alpha \neq 0 \Rightarrow$ ربع ۳ و ۴

$\frac{1-(1+\sin\alpha)}{|\cos|} = \tan\alpha \qquad \frac{1-1-\sin\alpha}{\cos\alpha} = \frac{\sin\alpha}{\cos\alpha}$

$\frac{-\sin\alpha}{|\cos\alpha|} = \frac{\sin\alpha}{\cos\alpha}$

$\frac{-1}{|\cos\alpha|} = \frac{1}{\cos\alpha} \Rightarrow \frac{|\cos\alpha|}{\cos\alpha} = -1$

↳ $\cos\alpha < 0$ ربع ۲ و ۳

= ربع ۳