۱)

$$\tan\frac{3\pi}{4}+\sin\frac{7\pi}{4}+\cos\frac{5\pi}{4}=\frac{\frac{\sqrt2}{2}}{-\frac{\sqrt2}{2}}-\frac{\sqrt2}{2}-\frac{\sqrt2}{2}=-1-\frac{2\sqrt2}{2}=-1-\sqrt2$$

$$\tan\frac{5\pi}{6}\times\sin\frac{5\pi}{3}+\cos\frac{4\pi}{3}=\frac{\frac12}{-\frac{\sqrt3}{2}}\times\frac{-\sqrt3}{2}+\frac{-1}{2}=\frac12\times\frac{-1}{\sqrt3}\times\frac{-\sqrt3}{2}=\frac12$$

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۲)

$$\frac{\cos x}{\sin x}=4 \qquad \cos x=4\sin x \qquad \frac{12\cos x-\sin x}{4\sin x+\sin x}=\frac{11\sin x}{5\sin x}=\frac{11}{5}$$

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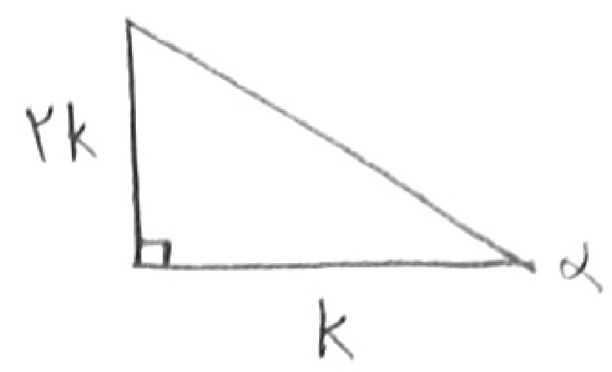
۳)

$$\cos\alpha\neq 0 \longrightarrow \sin\alpha=2\cos\alpha$$

$$\tan\alpha=2$$

ربع سوم $\begin{cases}\sin\alpha<0\\ \cos\alpha<0\\ \tan\alpha>0\end{cases}$



وتر $=\sqrt{(2k)^2+(k)^2}=\sqrt{5k^2}=k\sqrt5$

$$\cos\alpha=\frac{k}{k\sqrt5}=\frac{1}{\sqrt5}\times\frac{\sqrt5}{\sqrt5}=\frac{\sqrt5}{5}$$

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۴) الف

$$\cos\alpha=-\sqrt{1-\sin^2\alpha}=-\sqrt{1-\frac{2}{100}}=-\sqrt{\frac{49}{50}}=-\frac{7}{\sqrt{50}}=-\frac{7}{5\sqrt2}\times\frac{\sqrt2}{\sqrt2}=-\frac{7\sqrt2}{10}$$

$$\cos\left(\frac{11\pi}{4}+\alpha\right)=\cos\frac{11\pi}{4}\times\cos\alpha-\sin\frac{11\pi}{4}\times\sin\alpha$$

$$\cos\frac{11\pi}{4}=\cos\frac{3\pi}{4}=-\frac{\sqrt2}{2} \qquad \sin\frac{11\pi}{4}=\sin\frac{3\pi}{4}=\frac{\sqrt2}{2}$$

$$\cos\left(\frac{11\pi}{4}+\alpha\right)=-\frac{\sqrt2}{2}\cos\alpha-\frac{\sqrt2}{2}\sin\alpha=-\frac{\sqrt2}{2}(\cos\alpha+\sin\alpha)$$

$$-\frac{\sqrt2}{2}\left(-\frac{7\sqrt2}{10}+\frac{\sqrt2}{10}\right)=-\frac{\sqrt2}{2}\left(-\frac{3\sqrt2}{5}\right)=\frac{6}{10}=\frac35$$

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۴) ب)

$\tan=\frac17 \longrightarrow$ مقابل $=1$, مجاور $=7$ وتر $=\sqrt{1^2+7^2}=\sqrt{50}=5\sqrt2$

$$\sin\alpha=\frac{1}{5\sqrt2}=\frac{\sqrt2}{10} \qquad \cos\alpha=\frac{7}{5\sqrt2}=\frac{7\sqrt2}{10}$$

$$\frac{13\pi}{4}\equiv\frac{5\pi}{4} \longrightarrow \sin\left(\frac{13\pi}{4}+\alpha\right)=\sin\left(\frac{5\pi}{4}+\alpha\right)$$

$$\sin\left(\frac{5\pi}{4}+\alpha\right)=\sin\left(\frac{5\pi}{4}\right)\times\cos\alpha+\cos\left(\frac{5\pi}{4}\right)\times\sin\alpha=-\frac{\sqrt2}{2}\cos\alpha-\frac{\sqrt2}{2}\sin\alpha=-\frac{\sqrt2}{2}(\cos\alpha+\sin\alpha)$$

$$\cos\alpha+\sin\alpha=\frac{7\sqrt2}{10}+\frac{\sqrt2}{10}=\frac{8\sqrt2}{10}=\frac{4\sqrt2}{5}$$

$$\sin\left(\frac{13\pi}{4}+\alpha\right)=-\frac{\sqrt2}{2}\times\frac{4\sqrt2}{5}=-\frac45$$

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۵)

$$\sin^2\alpha+\sin^2\alpha+\cos^2\alpha=\frac43$$

$$\sin^2\alpha+1=\frac43 \qquad \sin^2\alpha=\frac43-\frac33=\frac13$$

$$\sin^2\alpha+\cos^2\alpha=1$$

$$\frac13+\cos^2\alpha=1$$

$$\cos^2\alpha=\frac23$$

$$\tan^2\alpha=\frac{\sin^2\alpha}{\cos^2\alpha}=\frac{\frac13}{\frac23}=\frac13\times\frac32=\frac12$$

(۶

$$S = \frac{1}{2}ab\sin\alpha = \frac{1}{2} \times \sqrt{2} \times 4\sqrt{2} \times \sin\alpha = 8\sin\alpha$$

$$2\sqrt{2}\sin\alpha = 8 \qquad \sin\alpha = \frac{8}{2\sqrt{2}} = \frac{\sqrt{2}}{2}$$

- $0 < \alpha < \pi \rightarrow \sin\alpha = \frac{\sqrt{2}}{2} \rightarrow \alpha_{min} = \frac{\pi}{2}$, $\alpha_{max} = \frac{2\pi}{2}$

$$\frac{a_{max}}{a_{min}} = \frac{\frac{2\pi}{2}}{\frac{\pi}{2}} = 2$$

(۷

$$S = ab\sin\alpha$$

$$a = 2k \quad b = 2k$$

$$\sin 30^\circ = \frac{1}{2}$$

$$8 = \cancel{2}k \times 2k \times \frac{1}{\cancel{2}} \Rightarrow 8 = 2k^2 \rightarrow k^2 = 16 \rightarrow k = \sqrt{16} = 2\sqrt{2}$$

$$a = 2k = 4\sqrt{2} \qquad b = 2k = 4\sqrt{2}$$

$$P = 2(a+b) = 2(4\sqrt{2} + 4\sqrt{2}) = 2(10\sqrt{2}) = 20\sqrt{2}$$

(۸

$$S_{ABC} = \frac{1}{2} \times 5 \times 7 \times \sin A = 17.5\sin A$$

$$S_{ADE} = \frac{1}{2} \times 7 \times 4 \times \sin A = 14\sin A$$

$$\tan 30^\circ = \frac{\frac{1}{2}}{\frac{\sqrt{2}}{2}} = \frac{1}{\cancel{2}} \times \frac{\cancel{2}}{\sqrt{2}} = \frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\left.\begin{array}{l} 17.5\sin A - 14\sin A = 2.5\sin A = 1.75 \\ \sin A = \frac{1.75}{2.5} = \frac{1}{2} \\ \hookrightarrow A = 30^\circ \end{array}\right\}$$

(۹

$$\frac{BN}{NC} = \frac{3}{2} \rightarrow BN = \frac{3}{5}BC \rightarrow \frac{BN}{BC} = \frac{3}{5}$$

$$\frac{S_{BMN}}{S_{ABC}} = \frac{BM}{BA} \times \frac{BN}{BC} \rightarrow \frac{BM}{BA} \times \frac{3}{5} = \frac{1}{3} \rightarrow \frac{BM}{BA} = \frac{5}{9}$$

$$AM = AB - BM$$

$$\frac{BM}{AM} = \frac{BM}{AB - BM} = \frac{\frac{5}{9}AB}{AB - \frac{5}{9}AB} = \frac{\frac{5}{9}}{\frac{4}{9}} = \frac{5}{9} \times \frac{9}{4} = \frac{5}{4}$$

(۱۰

$$\frac{1}{\sqrt{\cos^2\alpha}} - \tan^2\alpha = \frac{1+\sin\alpha}{|\cos\alpha|}$$

$$\sqrt{\cos^2\alpha} = |\cos\alpha| \longrightarrow \frac{1}{|\cos\alpha|} - \tan\alpha = \frac{1+\sin\alpha}{|\cos\alpha|}$$

$$\frac{1}{|\cos\alpha|} - \frac{\sin\alpha}{\cos\alpha} = \frac{1+\sin\alpha}{|\cos\alpha|}$$

if $\cos\alpha > 0 \longrightarrow |\cos\alpha| = \cos\alpha$

$$\frac{1+\sin\alpha}{\cos\alpha} : \text{right} \qquad \frac{1-\sin\alpha}{\cos\alpha} : \text{left}$$

$$1 - \sin\alpha = 1 + \sin\alpha \rightarrow \sin\alpha = 0$$

if $\cos\alpha < 0 \longrightarrow |\cos\alpha| = -\cos\alpha$

$$\frac{1+\sin\alpha}{|\cos\alpha|} = \frac{1+\sin\alpha}{-\cos\alpha} = -\frac{1+\sin\alpha}{\cos\alpha} : \text{right}$$

$$\frac{1}{-\cos\alpha} - \frac{\sin\alpha}{\cos\alpha} = -\frac{1}{\cos\alpha} - \frac{\sin\alpha}{\cos\alpha} = -\frac{1+\sin\alpha}{\cos\alpha} : \text{left}$$

$\sin 2 = 0$

$$\frac{|\sin\alpha|}{\cos\alpha} = -\frac{1}{\cot\alpha} = -\frac{\sin\alpha}{\cos\alpha}$$

$$\hookrightarrow |\sin\alpha| = -\sin\alpha$$

$\times \cos\alpha \neq 0$

$\sin\alpha \leq 0$

$\hookrightarrow \sin\alpha = 0$ ✓ $\rightarrow \alpha = 0, \pi, 2\pi$

انتهای کمان روی محور افقی $\begin{cases} \times \text{ مثبت} & \text{روی } \alpha = 0 \\ \times \text{ منفی} & \text{روی } \alpha = \pi \end{cases}$