دهم ریاضی A ۱۸

۱۸/۷۵

$-۱ \quad -\frac{\sqrt{۲}}{۲} \quad -\frac{\sqrt{۲}}{۲} \qquad -۱+\left(-\frac{\sqrt{۲}}{۲}\right)^۲ = -\frac{۱}{۲}$

۱)

$$\cancel{\tan\frac{۳\pi}{۴}} + \cancel{\sin\frac{۷\pi}{۴}} + \cancel{\cos\frac{۵\pi}{۴}} = \frac{\frac{\sqrt{۲}}{۲}}{-\frac{\sqrt{۲}}{۲}} - \frac{\sqrt{۲}}{۲} - \frac{\sqrt{۲}}{۲} = -۱-\frac{۲\sqrt{۲}}{۲} \cancel{= -۱-\sqrt{۲}}$$

۱/۷۵

$$\tan\frac{۵\pi}{۶} \times \sin\frac{۵\pi}{۳} + \cos\frac{۴\pi}{۳} = \frac{\frac{۱}{۲}}{-\frac{\sqrt{۳}}{۲}} \times \frac{-\sqrt{۳}}{۲} + \frac{-۱}{۲} = \frac{۱}{۲} \times \frac{\overset{-۱}{\cancel{۲}}}{\cancel{\sqrt{۳}}} \times \frac{\overset{-۱}{\cancel{-\sqrt{۳}}}}{\cancel{۲}} = \frac{۱}{۲}$$

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۲)

$$\frac{\cos x}{\sin x} = ۴ \qquad \cos x = ۴\sin x \qquad \frac{۱۲\cos x - \sin x}{۴\sin x + \sin x} = \frac{۱۱\sin x}{۵\sin x} = \frac{۱۱}{۵}$$

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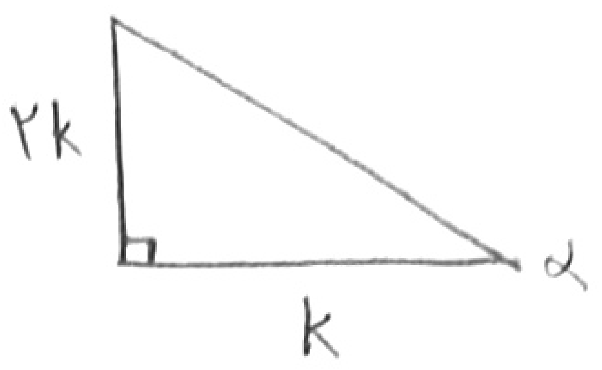
۳)

$$\cos\alpha \neq ۰ \longrightarrow \sin\alpha = ۲\cos\alpha$$

$$\tan\alpha = ۲$$

ربع سوم
$$\begin{cases} \sin\alpha < ۰ \\ \cos\alpha < ۰ \\ \tan\alpha > ۰ \end{cases}$$



$$\text{وتر} = \sqrt{(۲k)^۲+(k)^۲} = \sqrt{۵k^۲} = k\sqrt{۵}$$

$$\cos\alpha = \frac{k}{k\sqrt{۵}} = \frac{۱}{\sqrt{۵}} \times \frac{\sqrt{۵}}{\sqrt{۵}} = \frac{\sqrt{۵}}{۵}$$

۲

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۴) الف

$$\cos\alpha = -\sqrt{۱-\sin^۲\alpha} = -\sqrt{۱-\frac{۲}{۱۰۰}} = -\sqrt{\frac{۴۹}{۵۰}} = -\frac{۷}{\sqrt{۵۰}} = -\frac{۷}{۵\sqrt{۲}} \times \frac{\sqrt{۲}}{\sqrt{۲}} = -\frac{۷\sqrt{۲}}{۱۰}$$

$$\cos\left(۱۱\frac{\pi}{۴}+\alpha\right) = \cos\frac{۱۱\pi}{۴} \times \cos\alpha - \sin\frac{۱۱\pi}{۴} \times \sin\alpha$$

$$\cos\frac{۱۱\pi}{۴} = \cos\frac{۳\pi}{۴} = -\frac{\sqrt{۲}}{۲} \qquad \sin\frac{۱۱\pi}{۴} = \sin\frac{۳\pi}{۴} = \frac{\sqrt{۲}}{۲}$$

$$\cos\left(\frac{۱۱\pi}{۴}+\alpha\right) = -\frac{\sqrt{۲}}{۲}\cos\alpha - \frac{\sqrt{۲}}{۲}\sin\alpha = -\frac{\sqrt{۲}}{۲}(\cos\alpha+\sin\alpha)$$

$$-\frac{\sqrt{۲}}{۲}\left(-\frac{۷\sqrt{۲}}{۱۰}+\frac{\sqrt{۲}}{۱۰}\right) = -\frac{\sqrt{۲}}{۲}\left(-\frac{۳\sqrt{۲}}{۵}\right) = \frac{۶}{۱۰} = \frac{۳}{۵}$$

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۴) ب)

$$\tan = \frac{۱}{۷} \longrightarrow \text{مقابل} = ۱ \;,\; \text{مجاور} = ۷ \qquad \text{وتر} = \sqrt{۱^۲+۷^۲} = \sqrt{۵۰} = ۵\sqrt{۲}$$

$$\sin\alpha = \frac{۱}{۵\sqrt{۲}} = \frac{\sqrt{۲}}{۱۰} \qquad \cos\alpha = \frac{۷}{۵\sqrt{۲}} = \frac{۷\sqrt{۲}}{۱۰}$$

$$\frac{۱۳\pi}{۴} \equiv \frac{۵\pi}{۴} \longrightarrow \sin\left(\frac{۱۳\pi}{۴}+\alpha\right) = \sin\left(\frac{۵\pi}{۴}+\alpha\right)$$

$$\sin\left(\frac{۵\pi}{۴}+\alpha\right) = \sin\left(\frac{۵\pi}{۴}\right) \times \cos\alpha + \cos\left(\frac{۵\pi}{۴}\right) \times \sin\alpha = -\frac{\sqrt{۲}}{۲}\cos\alpha - \frac{\sqrt{۲}}{۲}\sin\alpha = -\frac{\sqrt{۲}}{۲}(\cos\alpha+\sin\alpha)$$

$$\cos\alpha+\sin\alpha = \frac{۷\sqrt{۲}}{۱۰} + \frac{\sqrt{۲}}{۱۰} = \frac{۸\sqrt{۲}}{۱۰} = \frac{۴\sqrt{۲}}{۵}$$

$$\sin\left(\frac{۱۳\pi}{۴}+\alpha\right) = -\frac{\sqrt{۲}}{\cancel{۲}} \times \frac{\overset{۲}{\cancel{۴}}\sqrt{۲}}{۵} = -\frac{۴}{۵}$$

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۵)

$$\sin^۲\alpha + \sin^۲\alpha + \cos^۲\alpha = \frac{۴}{۳}$$

$$\sin^۲\alpha + ۱ = \frac{۴}{۳} \qquad \sin^۲\alpha = \frac{۴}{۳} - \frac{۳}{۳} = \frac{۱}{۳}$$

$$\sin^۲\alpha + \cos^۲\alpha = ۱$$

$$\frac{۱}{۳} + \cos^۲\alpha = ۱$$

$$\cos^۲\alpha = \frac{۲}{۳}$$

$$\tan^۲\alpha = \frac{\sin^۲\alpha}{\cos^۲\alpha} = \frac{\frac{۱}{۳}}{\frac{۲}{۳}} = \frac{۱}{\cancel{۳}} \times \frac{\cancel{۳}}{۲} = \frac{۱}{۲}$$

(۶

$S=\frac{1}{۲}ab\sin\alpha = \frac{1}{\cancel{۲}}\times\sqrt{۳}\times\overset{۳}{۶}\times\sin\alpha = ۴٫۵$

$۳\sqrt{۳}\sin\alpha = ۴٫۵ \qquad \sin\alpha = \frac{۴٫۵}{۳\sqrt{۳}} = \frac{\sqrt{۳}}{۲}$

$۰<\alpha<\pi \rightarrow \sin\alpha = \frac{\sqrt{۳}}{۲} \rightarrow \alpha_{min} = \frac{\pi}{۳} \,,\, \alpha_{max} = \frac{۲\pi}{۳}$

$\frac{a\,max}{a\,min} = \frac{\frac{۲\pi}{۳}}{\frac{\pi}{۳}} = ۲$

ص

(۷

$S = ab\sin\alpha$

$a=۲k \quad b=۳k$

$\sin ۱۵۰^\circ = \frac{1}{۲}$

$۵۴ = \cancel{۲}k\times ۳k\times\frac{1}{\cancel{۲}} \Rightarrow ۵۴ = ۳k^۲ \rightarrow k^۲ = ۱۸ \rightarrow k = \sqrt{۱۸} = ۳\sqrt{۲}$

$a = ۲k = ۶\sqrt{۲} \qquad b = ۳k = ۹\sqrt{۲}$

$P = ۲(a+b) = ۲(۶\sqrt{۲}+۹\sqrt{۲}) = ۲(۱۵\sqrt{۲}) = ۳۰\sqrt{۲}$

ص

(۸

$S_{ABC} = \frac{1}{۲}\times ۵\times ۷\times\sin_A = ۱۷٫۵\sin_A$

$S_{ADE} = \frac{1}{۲}\times ۷\times ۴\times\sin_A = ۱۴\sin A$

$۱۷٫۵\sin_A - ۱۴\sin_A = ۳٫۵\sin A = ۱٫۷۵$

$\sin A = \frac{۱٫۷۵}{۳٫۵} = \frac{1}{۲}$

$\rightarrow A = ۳۰^\circ$

$\tan ۳۰^\circ = \frac{\frac{1}{۲}}{\frac{\sqrt{۳}}{۲}} = \frac{1}{\cancel{۲}}\times\frac{\cancel{۲}}{\sqrt{۳}} = \frac{1}{\sqrt{۳}}\times\frac{\sqrt{۳}}{\sqrt{۳}} = \frac{\sqrt{۳}}{۳}$

ص

(۹

$\frac{BN}{NC} = \frac{۳}{۲} \rightarrow BN = \frac{۳}{۵}BC \rightarrow \frac{BN}{BC} = \frac{۳}{۵}$

ص

$\frac{BMN}{ABC} = \frac{BM}{BA}\times\frac{BN}{BC} \longrightarrow \frac{BM}{BA}\times\frac{۳}{۵} = \frac{1}{۳} \longrightarrow \frac{BM}{BA} = \frac{۵}{۹}$

$AM = AB - BM$

$\frac{BM}{AM} = \frac{BM}{AB-BM} = \frac{\frac{۵}{۹}AB}{AB-\frac{۵}{۹}AB} = \frac{\frac{۵}{۹}}{\frac{۴}{۹}} = \frac{۵}{۹}\times\frac{۹}{۴} = \frac{۵}{۴}$

(۱۰

$\frac{1}{\sqrt{\cos^۲\alpha}} - \tan^۲\alpha = \frac{1+\sin\alpha}{|\cos\alpha|}$

$\sqrt{\cos^۲\alpha} = |\cos\alpha| \longrightarrow \frac{1}{|\cos\alpha|} - \tan\alpha = \frac{1+\sin\alpha}{|\cos\alpha|}$

$\frac{1}{|\cos\alpha|} - \frac{\sin\alpha}{\cos\alpha} = \frac{1+\sin\alpha}{|\cos\alpha|}$

~~$\text{if } \cos\alpha>۰ \longrightarrow |\cos\alpha| = \cos\alpha$~~

~~$\frac{1+\sin\alpha}{\cos\alpha}$: right $\quad \frac{1-\sin\alpha}{\cos\alpha}$: left~~

~~$1-\sin\alpha = 1+\sin\alpha \rightarrow \sin\alpha = ۰$~~

~~$\text{if } \cos<۰ \rightarrow |\cos\alpha| = -\cos\alpha$~~

~~$\frac{1+\sin\alpha}{|\cos\alpha|} = \frac{1+\sin\alpha}{-\cos\alpha} = -\frac{1+\sin\alpha}{\cos\alpha}$: right~~

~~$\frac{1}{-\cos\alpha} - \frac{\sin\alpha}{\cos\alpha} = -\frac{1}{\cos\alpha} - \frac{\sin\alpha}{\cos\alpha} = -\frac{1+\sin\alpha}{\cos\alpha}$: left~~

$\sin\alpha = ۰$

$\frac{|\sin\alpha|}{\cos\alpha} = -\frac{1}{\cot\alpha} = -\frac{\sin\alpha}{\cos\alpha}$

$\xrightarrow[\times\cos\alpha\neq۰]{} |\sin\alpha| = -\sin\alpha$ ①

$\rightarrow \sin\alpha < ۰$

$\sin\alpha \leq ۰$

$\rightarrow \sin\alpha = ۰ \; ✓ \rightarrow \alpha = ۰, \pi, ۲\pi$

انتهای کمان روی محور افقی {
α = ۰ روی محور مثبت x
α = π روی محور منفی x

$\rightarrow \frac{1}{|\cos\alpha|} - \frac{1+\sin\alpha}{|\cos\alpha|} = +\frac{\sin\alpha}{\cos\alpha}$

$\frac{-\sin\alpha}{|\cos\alpha|} = +\frac{\sin\alpha}{\cos\alpha}$

①

② $\cos\alpha < ۰$

$۱ ، ۲ \rightarrow$ α ربع ۳ است