رقم دفتر A

عارفه سعادت نیا — ۲۵

۱)

$\frac{11\pi}{4} \times \frac{180}{\pi} = 495 - 360 = 135$ $\quad \tan 135^\circ = -1$ $\quad \frac{15\pi}{4} \times \frac{180}{\pi} = 675 - 360 = 315$

$\frac{13\pi}{4} \times \frac{180}{\pi} = 585 - 360 = 225$ $\quad \cos 225^\circ = -\frac{\sqrt{2}}{2}$ $\quad \sin 315^\circ = -\frac{\sqrt{2}}{2}$

الف ، $-1 \cdot \left(-\frac{\sqrt{2}}{2} \times -\frac{\sqrt{2}}{2}\right) = -\frac{1}{2}$

ب ، $\frac{17\pi}{6} \times \frac{180}{\pi} = 510 - 360 = 150$ $\quad \tan 150^\circ = -\frac{\sqrt{3}}{3}$ $\quad \frac{11\pi}{3} \times \frac{180}{\pi} = 660 - 360 = 300$

$\frac{10\pi}{3} \times \frac{180}{\pi} = 600 - 360 = 240$ $\quad \cos 240^\circ = -\frac{1}{2}$ $\quad \sin 300^\circ = -\frac{\sqrt{3}}{2}$

$\left(-\frac{\sqrt{3}}{3} \cdot -\frac{\sqrt{3}}{2}\right) - \frac{1}{2} = \frac{1}{2} - \frac{1}{2} = 0$

۲)

$$\frac{3\cos x - \sin x}{\cos x + \sin x} \xrightarrow{\div \sin x} \frac{\frac{3\cos x}{\sin x} - \frac{\sin x}{\sin x}}{\frac{\cos x}{\sin x} + \frac{\sin x}{\sin x}} = \frac{3\cot x - 1}{\cot x + 1} = \frac{12 - 1}{5} = \frac{11}{5}$$

۳)

$\tan\alpha = \frac{\sin\alpha}{2\sin\alpha} = 2$ $\quad \cos^2\alpha = \frac{1}{1+\tan^2\alpha} = \frac{1}{1+4} = \frac{1}{5} \to \cos\alpha = -\frac{\sqrt{5}}{5}$

۴)

الف ، $\sin^2 + \cos^2\alpha = 1 \to \cos^2\alpha = 1 - \sin^2\alpha \to \cos^2\alpha = 1 - \frac{2}{100} = \frac{98}{100} = \frac{49}{50} \to \cos\alpha = \frac{7}{\sqrt{50}} = 5\sqrt{2}$

$\cos\alpha = \frac{-7}{5\sqrt{2}} = \frac{-7\sqrt{2}}{10} \to \cos\frac{11\pi}{4} = -\frac{\sqrt{2}}{2} \to \cos\left(\frac{11\pi}{4} + \alpha\right) = \left(-\frac{7\sqrt{2}}{10}\right)\left(-\frac{\sqrt{2}}{2}\right) - \left(\frac{\sqrt{2}}{10}\right)\left(\frac{\sqrt{2}}{2}\right) =$

$4\pi = \frac{8\pi}{4} \leftrightarrow \frac{8\pi}{4} + \frac{3\pi}{4} \to \frac{11\pi}{4} = \frac{3\pi}{4} \to \cos\frac{11\pi}{4} = \cos\frac{3\pi}{4}$ $\quad \frac{14}{20} - \frac{2}{20} = \frac{12}{20} = \frac{3}{5}$

ب ، $\sin(\alpha + \beta) = \sin\alpha\cos\beta + \sin\beta\cos\alpha$ $\quad 1 + \tan^2\alpha = \frac{1}{\cos^2\alpha} \to 1 + \frac{1}{49} \to \frac{49}{50} = \cos^2\alpha \to \cos\alpha:$

$\sin^2\alpha = 1 - \cos^2\alpha \to 1 - \frac{49}{50} = \frac{1}{50} = \sin^2\alpha \to \sin\alpha = \pm\frac{1}{5\sqrt{2}} = \pm\frac{\sqrt{2}}{10}$ $\quad \pm\frac{7}{5\sqrt{2}}$

$\sin\left(\frac{13\pi}{4} + \alpha\right) = \left(-\frac{\sqrt{2}}{2}\right)\left(\frac{7\sqrt{2}}{10}\right) + \left(\frac{\sqrt{2}}{10}\right)\left(-\frac{\sqrt{2}}{2}\right) = \frac{-14}{20} + \left(\frac{-2}{20}\right) = \frac{-16}{20} = -\frac{4}{5}$

$3\sin^2 x + \cos^2 x = \frac{5}{4}$ ⑤

$\sin^2 x = 1 - \cos^2 x \rightarrow 3(1-\cos^2 x) + \cos^2 x = \frac{5}{4} \rightarrow -\cos^2 x + 3 = \frac{5}{4} \rightarrow \cos^2 x = -\frac{7}{4}$

$\cos^2 x = \frac{3}{4}$, $\sin^2 x = 1 - \cos^2 x = 1 - \frac{3}{4} = \frac{1}{4}$ $\tan^2 x = \frac{\frac{1}{4}}{\frac{3}{4}} = \frac{1}{3}$

$S = \frac{1}{2} ab \sin\alpha \rightarrow \frac{1}{2} \times \sqrt{3} \times 4 \times \sin\alpha = 6\sqrt{3} \rightarrow 2\sqrt{3} \times \sin\alpha = 6\sqrt{3}$ ④

$\sin\alpha = \frac{6\sqrt{3}}{2\sqrt{3}} = \frac{\sqrt{3}}{2} \rightarrow \alpha = 60^\circ \lor 120^\circ$ $\frac{120}{60} = 2$

⑦

$\frac{a}{b} = \frac{2}{3}$ $S = ab\sin\alpha = 96 \rightarrow (2k)(3k)(\frac{1}{2}) = 96 \rightarrow 6k^2 = 96 \times 2 = 192$

$k^2 = 16 \rightarrow k = 4$

$a = 2(4\sqrt{2}) = 8\sqrt{2}$, $b = 3(4\sqrt{2}) = 12\sqrt{2}$ $\rightarrow$ محیط $= 2(a+b) = 2(8\sqrt{2}+12\sqrt{2}) = 40\sqrt{2}$

⑧

$S_{ABC} - S_{ADE} = 1,75$

$(\frac{1}{2} \times 10 \times 7 \sin A) - (\frac{1}{2} \times 4 \times 7 \times \sin\alpha) = 1,75 \rightarrow \frac{35}{2}\sin\alpha - 14\sin\alpha = 17,5\alpha = 1,75$

$\sin\alpha = \frac{1,75}{17,5} = \frac{1}{10}$ مثلث $\cos\alpha = \frac{\sqrt{3}}{2} \rightarrow \alpha = 30^\circ$ $\tan\alpha = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{\sqrt{3}}{3}$

⑨

$S_{ABC} = \frac{1}{2} \times \overline{AB} \times \overline{BC} \times \sin B = 2(\frac{1}{2} \times \overline{BM} \times \overline{BN} \times \sin B)$

$(\overline{AB} \times \overline{BC}) = 2 \times \overline{BM} \times \overline{BN} \rightarrow (\overline{AM} + \overline{BM})(\overline{BN} + \overline{NC}) = 2(\overline{BM} \times \overline{BN})$

$\frac{5}{2}\overline{BN}(\overline{AM} + \overline{BM}) = 2 \times \overline{BM} \times \overline{BN} \rightarrow \frac{5}{2}(\overline{AM} + \overline{BM}) = 2\overline{BM} \rightarrow (\frac{5}{2}\overline{AM} = \frac{3}{2}\overline{BM})$

$5\overline{AM} = 3\overline{BM} \rightarrow \overline{BM} = \frac{5}{3}\overline{AM}$ $\quad \ll \frac{BM}{AM} = \frac{5}{3} \gg$

⑩

$\frac{1}{\sqrt{\cos^2\alpha}} \rightarrow \tan\alpha = \frac{1+\sin\alpha}{|\cos\alpha|} \cdot \frac{|\sin\alpha|}{\cos\alpha} = (-\frac{1}{\cot\alpha}) \rightarrow |\sin\alpha| = -\sin\alpha \rightarrow \sin\alpha < 0$ ①

$\frac{1}{|\cos\alpha|} \cdot \frac{\sin\alpha}{\cos\alpha} = \frac{1-\sin^2}{\cos\alpha}$ if $\cos > 0$ $\quad -\frac{\sin\alpha}{\cos\alpha}$

$\frac{1-\sin\alpha}{\cos\alpha} \rightarrow \frac{1+\sin\alpha}{\cos\alpha} \rightarrow 1 = \sin\alpha = 1 + \sin\alpha \rightarrow \sin\alpha = 0 \rightarrow \sin < 0$ ✗

if $\cos < 0$ $\rightarrow$ $\frac{1}{-\cos\alpha} + \frac{\sin\alpha}{-\cos\alpha} = \frac{1+\sin\alpha}{-\cos\alpha} \rightarrow \frac{1+\sin\alpha}{-\cos\alpha} = \frac{1+\sin\alpha}{|\cos\alpha|} \rightarrow$ ④ $\cos\alpha < 0$ ✓

$\sin < 0 \rightarrow \cos < 0 \rightarrow$ (ربع ۳)