

۱

الف) $\tan \frac{11\pi}{6} + \sin \frac{10\pi}{6} \cos \frac{13\pi}{6} \Rightarrow -1 - \frac{\sqrt{3}}{2} \times (-\frac{\sqrt{3}}{2}) = -1 + \frac{1}{2} = \boxed{-\frac{1}{2}}$

$\frac{11\pi}{6} + \frac{\pi}{6} \Rightarrow \tan = -1$ $\frac{10\pi}{6} + \frac{2\pi}{6} \Rightarrow \sin = -\frac{\sqrt{3}}{2}$ $\frac{13\pi}{6} + \frac{5\pi}{6} \Rightarrow \cos = -\frac{\sqrt{3}}{2}$ $+\frac{1}{2} = +\frac{1}{2}$

ب) $\tan \frac{17\pi}{6} \sin \frac{11\pi}{6} + \cos \frac{10\pi}{6} \Rightarrow -\frac{1}{\sqrt{3}} \times (-\frac{\sqrt{3}}{2}) - \frac{1}{2} = \boxed{0}$

$\frac{17\pi}{6} + \frac{5\pi}{6} \Rightarrow \tan = -\frac{1}{\sqrt{3}}$ $\frac{11\pi}{6} + \frac{2\pi}{6} \Rightarrow \sin = -\frac{\sqrt{3}}{2}$ $\frac{10\pi}{6} + \frac{2\pi}{6} \Rightarrow \cos = -\frac{1}{2}$

$\frac{3\cos x - \sin x}{\cos x + \sin x} \Rightarrow \frac{\frac{3}{\sin x} - \frac{1}{\sin x}}{\frac{3}{\sin x} + \frac{1}{\sin x}} = \frac{14-1}{14+1} = \boxed{\frac{11}{15}}$

$\cot x = 14$

۲

$\sin^2 x + \cos^2 x = 1$

$\sin x = 2\cos x$

۳

$(2\cos x)^2 + \cos^2 x = 1 \Rightarrow 4\cos^2 x + \cos^2 x = 1$

$5\cos^2 x = 1 \Rightarrow \cos x = \pm \frac{1}{\sqrt{5}}$

چون در ربع سوم سینوس منفی است $\Rightarrow \cos x = -\frac{1}{\sqrt{5}}$

الف) $\sin x = \frac{\sqrt{3}}{10}$ $\cos(\frac{11\pi}{6} + x)$ $\cos(\beta + \alpha) = \cos \beta \cos \alpha - \sin \beta \sin \alpha = \frac{\sqrt{3}}{2} \times (-\frac{1}{2\sqrt{5}}) - (\frac{\sqrt{3}}{2}) \times \frac{\sqrt{3}}{10} = -\frac{\sqrt{3}}{10} - \frac{3}{20} = \boxed{-\frac{5\sqrt{3} + 3}{20}}$

$\sin^2 x + \cos^2 x = 1$ $\frac{3}{100} + \cos^2 x = 1 \Rightarrow \cos^2 x = \frac{97}{100} \Rightarrow \cos x = \pm \frac{\sqrt{97}}{10}$

اینجا چون α در ربع دوم سینوس مثبت است $\Rightarrow \cos x = -\frac{\sqrt{97}}{10}$

۴

ب) $\tan x = \frac{1}{\sqrt{3}}$ $\sin(\frac{13\pi}{6} + x)$ $\sin(\beta + \alpha) = \sin \beta \cos \alpha + \sin \alpha \cos \beta = -\frac{\sqrt{3}}{2} \times \frac{1}{2\sqrt{3}} + \frac{1}{2} \times (-\frac{\sqrt{3}}{2}) = -\frac{1}{4} - \frac{\sqrt{3}}{4} = \boxed{-\frac{1+\sqrt{3}}{4}}$

$\cos \frac{13\pi}{6} = -\frac{\sqrt{3}}{2}$ $\sin \frac{13\pi}{6} = -\frac{1}{2}$

$1 + \tan^2 x = \frac{1}{\cos^2 x} \Rightarrow 1 + \frac{1}{3} = \frac{1}{\cos^2 x} \Rightarrow \frac{4}{3} = \frac{1}{\cos^2 x} \Rightarrow \cos^2 x = \frac{3}{4} \Rightarrow \cos x = \pm \frac{\sqrt{3}}{2}$

چون اینجا α در ربع اول سینوس مثبت است $\Rightarrow \cos x = \frac{\sqrt{3}}{2}$

$\sin x = \frac{1}{2\sqrt{3}}$ $\frac{\sin x}{\cos x} = \frac{1}{\sqrt{3}}$

$3\sin^2 x + \cos^2 x = \frac{4}{3}$ $\tan^2 x = ?$ $\tan^2 x = \frac{\sin^2 x}{\cos^2 x} = \frac{\frac{1}{3}}{\frac{1}{3}} = \boxed{1}$

$\cos^2 x = 1 - \sin^2 x = \frac{1}{3}$

۵

$3\sin^2 x + 1 - \sin^2 x = \frac{4}{3}$ $2\sin^2 x = \frac{4}{3} - 1 \Rightarrow \sin^2 x = \frac{1}{3} \Rightarrow \sin x = \pm \frac{1}{\sqrt{3}}$

$$S_{ABC} = \frac{1}{2} \times a \times b \times \sin \alpha$$

$$\frac{\sin \alpha}{\sin \beta} = \frac{12}{9} = 4/3$$

نسبت ۱۲ و ۹ را می توانیم ساده کنیم

$$\frac{1}{2} \times \sqrt{13} \times \frac{1}{2} \times \sin \alpha = 15/2$$

$$\sqrt{13} \times \sin \alpha = 15$$

$$\sin \alpha = \frac{15}{\sqrt{13}} = \frac{15}{\sqrt{13}} \times \frac{\sqrt{13}}{\sqrt{13}} = \frac{15\sqrt{13}}{13}$$

$$\frac{1}{2} \times 9 \times b \times \sin \alpha = 15$$

$$a \times b = 10$$

$$\sin 10^\circ = \frac{1}{2}$$

$$\frac{a}{b} = \frac{9}{10}$$

$$\frac{9b}{10} \times b = 10$$

$$\frac{9b^2}{10} = 10 \Rightarrow b^2 = 100/9 \Rightarrow b = \sqrt{100/9} = 10/3$$

$$S_{ABC} = \frac{1}{2} (9 \times \frac{10}{3} \times \sin 10^\circ) = 15 \sin 10^\circ$$

$$a = \frac{9 \times \frac{10}{3}}{10} = 3$$

$$S_{ABC} = \frac{1}{2} \times AB \times AC \times \sin A \Rightarrow \frac{1}{2} \times d \times V \times \sin A = \frac{15}{2} \sin A$$

$$S_{ADE} = \frac{1}{2} \times V \times F \times \sin A = 15 \sin A$$

$$\Rightarrow \frac{15}{2} \sin A - (15 \sin A) = \sin A (15/2 - 15) = -15/2 \sin A$$

$$\frac{15}{2} \sin A = 15/2$$

$$\sin A = 1/2 \Rightarrow A = 30^\circ$$

$$\tan 30^\circ = \frac{1}{\sqrt{3}}$$

$$\frac{S_{BMN}}{S_{ABC}} = \frac{BM \times BN}{BA \times BC} = \frac{BM}{AB} \times \frac{1}{2} = \frac{1}{2}$$

$$\frac{BM}{AB} = \frac{1}{2} \Rightarrow \frac{BM}{AM} = \frac{1}{1}$$

$$\frac{1}{\sqrt{\cos^2 \alpha}} - \tan \alpha = \frac{1 + \sin \alpha}{|\cos \alpha|} \Rightarrow \frac{1}{|\cos \alpha|} - \tan \alpha = \frac{1 + \sin \alpha}{|\cos \alpha|}$$

$$|\cos \alpha|$$

$$\frac{1}{|\cos \alpha|} - \frac{\sin \alpha}{\cos \alpha} = \frac{1 + \sin \alpha}{|\cos \alpha|}$$

$$1 - \sin \alpha = 1 + \sin \alpha$$

$$-\sin \alpha = \sin \alpha$$

$$-2 \sin \alpha = 0$$

$$\sin \alpha = 0$$

$$\frac{|\sin \alpha|}{\cos \alpha} = -\frac{1}{\cot \alpha}$$

$$\frac{|\sin \alpha|}{\cos \alpha} = -\frac{\sin \alpha}{\cos \alpha}$$

$$|\sin \alpha| = -\sin \alpha$$

$$\sin \alpha \leq 0$$

نتیجه این که در ربع سوم و چهارم قرار می گیرد