سارا عزیزی

۱) الف) $\tan\frac{11\pi}{4}+\sin\frac{15\pi}{4}\cos\frac{12\pi}{4}=\tan\left(\frac{8\pi}{4}+\frac{3\pi}{4}\right)+\sin\left(\frac{12\pi}{4}+\frac{3\pi}{4}\right)\times\cos\left(\frac{16\pi}{4}+\frac{3\pi}{4}\right)$

$=-\tan^2\frac{\pi}{4}+\left(+\sin^2\frac{\pi}{4}\times+\cos\frac{\pi}{4}\right)=-\tan^2\frac{\pi}{4}+\sin^2\frac{\pi}{4}\cos\frac{\pi}{4}=-(-1)+\frac{1}{2}\times\frac{\sqrt{2}}{2}$

$=1+\frac{1}{4}=\frac{5}{4}$

ب) $\tan\frac{17\pi}{3}\sin\frac{11\pi}{3}+\cos\frac{10\pi}{3}=\tan\left(\frac{12\pi}{3}+\frac{5\pi}{3}\right)\times\sin\left(\frac{9\pi}{3}+\frac{2\pi}{3}\right)+\cos\left(\frac{9\pi}{3}+\frac{\pi}{3}\right)$

$=-\tan\frac{5\pi}{3}\times-\sin\frac{2\pi}{3}+\cos\frac{\pi}{3}=-\left(-\frac{1}{\sqrt{3}}\right)\times-\left(\frac{\sqrt{3}}{2}\right)+\frac{1}{2}=\frac{1}{\sqrt{3}}\times-\frac{\sqrt{3}}{2}+\frac{1}{2}$

$=-\frac{1}{2}+\frac{1}{2}=0$

۲) $\cot\alpha=4\rightarrow\frac{\cos\alpha}{\sin\alpha}=4\rightarrow\cos\alpha=4\sin\alpha$

$\frac{4\cos\alpha-\sin\alpha}{\cos\alpha+\sin\alpha}=\frac{4(4\sin\alpha)-\sin\alpha}{4\sin\alpha+\sin\alpha}=\frac{11\sin\alpha}{5\sin\alpha}=\boxed{\frac{11}{5}}$

۳) $\sin\alpha=2\cos\alpha$, $\sin\alpha<0$, $\cos\alpha<0$ $\quad\cos\alpha=?$

$\sin^2\alpha+\cos^2\alpha=1\rightarrow(2\cos\alpha)^2+\cos^2\alpha=1\rightarrow4\cos^2\alpha+\cos^2\alpha=5\cos^2\alpha=1$

$\rightarrow\cos^2\alpha=\frac{1}{5}\rightarrow\cos\alpha=\frac{1}{\sqrt{5}}\times\frac{\sqrt{5}}{\sqrt{5}}=\frac{\sqrt{5}}{5}\rightarrow\boxed{\cos\alpha=-\frac{\sqrt{5}}{5}}$

۴) الف) $\sin\alpha>0$, $\cos\alpha<0$, $\sin\alpha=\frac{\sqrt{21}}{5}\rightarrow\cos\alpha=\sqrt{1-\frac{21}{25}}\rightarrow\cos\alpha=\sqrt{\frac{4}{25}}=-\frac{2\sqrt{21}}{5}$

$\cos\left(\frac{11\pi}{4}+\alpha\right)=\left(\cos\frac{11\pi}{4}\times\cos\alpha\right)-\left(\sin\frac{11\pi}{4}\times\sin\alpha\right)=-\frac{\sqrt{2}}{2}\cos\alpha-\frac{\sqrt{2}}{2}\sin\alpha$

$=\left(+\frac{\sqrt{2}}{2}\times\frac{2\sqrt{21}}{5}\right)-\left(\frac{\sqrt{2}}{2}\times\frac{\sqrt{21}}{5}\right)=\frac{2}{5}-\frac{1}{5}=\boxed{\frac{1}{5}}$

ب) $\sin\alpha>0$, $\cos\alpha>0$, $\tan\alpha=\frac{1}{2}\rightarrow\frac{1}{4}+1=\frac{5}{4}=\frac{1}{\cos^2\alpha}\rightarrow\cos^2\alpha=\frac{4}{5}\rightarrow\cos\alpha=\frac{2}{\sqrt{5}}$

$\rightarrow\cos^2\alpha+\sin^2\alpha=\frac{4}{5}+\sin^2\alpha=1\rightarrow\sin^2\alpha=\frac{1}{5}\rightarrow\sin\alpha=\frac{1}{\sqrt{5}}$

$\sin\left(\frac{13\pi}{4}+\alpha\right)=\left(\cos\frac{13\pi}{4}\times\sin\alpha\right)+\left(\sin\frac{12\pi}{4}\times\cos\alpha\right)=\left(-\frac{\sqrt{2}}{2}\times\frac{1}{\sqrt{5}}\right)+\left(-\frac{\sqrt{2}}{2}\times\frac{2}{\sqrt{5}}\right)$

$=-\frac{1}{10}-\frac{2}{10}=-\frac{3}{10}$

(۳

$۲\sin^۲x + \cos^۲x = \frac{۶}{۴} \to \sin^۲x + ۲\sin^۲x\cos^۲x = \frac{۱}{۲} + ۱ \to \sin^۲x = \frac{۱}{۴}$

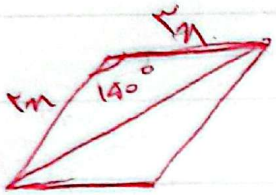
$\to \frac{۱}{۴} + \cos^۲x = ۱ \to \cos^۲x = \frac{۳}{۴} \to \tan^۲x = \frac{\sin^۲x}{\cos^۲x} = \left(\frac{\frac{۱}{۴}}{\frac{۳}{۴}} = \boxed{\frac{۱}{۳}}\right)$

(۴

$S_{ABC} = \frac{۱}{۲} \times \sqrt{۲} \times ۲ \times \sin\alpha = \frac{۲}{۲} \times \frac{۱}{۲} \to ۲\sqrt{۲}\sin\alpha = ۲ \to \sin\alpha = \frac{۲}{۲\sqrt{۲}} \times \frac{\sqrt{۲}}{\sqrt{۲}}$

$= \frac{\sqrt{۲}}{۲} \to \alpha_{max} = ۱۳۰^\circ \; , \alpha_{min} = ۵۰^\circ \to \frac{۱۳۰}{۵۰} = \boxed{۲}$

(۷



$S = ۵\varepsilon = \frac{۲}{۲} \times \frac{۱}{۲} \times ۲n \times ۲n \times \frac{۱}{۲} \to ۵\varepsilon = ۲n^۲$

$\to ۵۱۸ = n^۲ \to n = \sqrt{۱۸} = ۳\sqrt{۲}$

$\to \frac{۱}{۲}\alpha = ۲\left(\frac{۹\sqrt{۲}}{۲} + \frac{۷\sqrt{۲}}{۲}\right) = \boxed{۲۰\sqrt{۲}}$

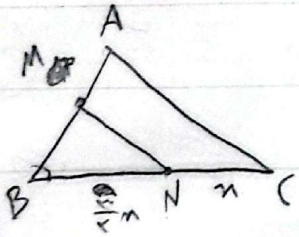
(۸

$S_{ABC} - S_{ABE} = ۱۷۵ \to ۱۷۵\sin A - ۱۴\sin A = ۳۷۵\sin A = ۱۷۵ \to \sin A = \frac{۱}{۳}$

$S_{ABE} = \frac{۱}{۲} \times ۵ \times ۷ \times \sin A = \frac{۳۵}{۲}\sin A = ۱۷۵\sin A \to S_{ADE} = \frac{۱}{۲} \times \frac{۲}{۵} \times ۷ \times \sin A = ۱۴\sin A$

$\to \frac{۱}{۲} + \cos^۲A = ۱ \to \cos^۲A = \frac{۲}{۳} \to \cos A = \frac{\sqrt{۲}}{۳} \to \tan A = \left(\frac{\frac{۱}{۳}}{\frac{\sqrt{۲}}{۳}} = \frac{۱}{\sqrt{۲}} \times \frac{\sqrt{۲}}{\sqrt{۲}} = \boxed{\frac{\sqrt{۲}}{۲}}\right)$

(۹



$۲BN = ۲NC \to BN = \frac{۲}{۲}NC$

$\frac{۱}{۲} AB \times BC \times \sin B = \frac{۱}{۲} \times BM \times BN \times \sin B \times ۲$

$AB \times BC = ۲ \times BM \times BN \to (BM + AM) \times (BN + NC) = ۲ \times BN \times BN$

$\to \frac{۵}{۲} \times (BM + AM) = ۲ \times BM \times \frac{۲}{۲} \times \frac{۲}{۲} \to ۵BM + ۵AM = ۹BM \to ۵AM = ۴BM$

$\to AM = \frac{۴}{۵}BM \to \frac{BM}{AM} = \frac{BM}{\frac{۴}{۵}BM} = \frac{۵}{۴}BM$

$\frac{۱}{\sqrt{\cos^۲\alpha}} - \tan\alpha = \frac{۱+\sin\alpha}{|\cos\alpha|}$, $\frac{|\sin\alpha|}{\cos\alpha} = -\frac{۱}{\cot\alpha} = -\frac{\sin\alpha}{\cos\alpha} \to \sin\alpha < ۰$ (۱۰

$\to \frac{۱}{|\cos\alpha|} - \frac{\sin\alpha}{\cos\alpha} = \frac{۱-\sin\alpha}{|\cos\alpha|} \to \frac{۱+\sin\alpha}{\cos\alpha} > ۰ \to \cos\alpha < ۰ \to \boxed{\text{ربع سوم}}$