۱۹/۵

سارا عزیزی

$\tan\frac{3\pi}{4} = -1$

۱) الف) $\tan\frac{11\pi}{4} + \sin\frac{15\pi}{4} \cos\frac{13\pi}{4} = \tan\left(\frac{8\pi}{4}+\frac{3\pi}{4}\right) + \sin\left(\frac{12\pi}{4}+\frac{3\pi}{4}\right)\times\cos\left(\frac{12\pi}{4}+\frac{\pi}{4}\right)$

$= -\tan\frac{3\pi}{4} + \left(+\sin\frac{3\pi}{4} \times +\cos\frac{\pi}{4}\right) = -\tan\frac{3\pi}{4} + \sin\frac{3\pi}{4}\cos\frac{\pi}{4} = -(-1) + \frac{\sqrt{2}}{2}\times\frac{\sqrt{2}}{2}$

۱/۵

$= -1 + \frac{1}{2} = \cancel{\frac{3}{2}}$ $\frac{-1}{2}$

ب) $\tan\frac{17\pi}{6}\sin\frac{11\pi}{3} + \cos\frac{10\pi}{3} = \tan\left(\frac{12\pi}{6}+\frac{5\pi}{6}\right)\times\sin\left(\frac{9\pi}{3}+\frac{2\pi}{3}\right)+\cos\left(\frac{9\pi}{3}+\frac{\pi}{3}\right)$

$= -\tan\frac{5\pi}{6} \times -\sin\frac{2\pi}{3} + \cos\frac{\pi}{3} = -\left(-\frac{1}{\sqrt{3}}\right) \times -\left(\frac{\sqrt{3}}{2}\right) + \frac{1}{2} = \frac{1}{\sqrt{3}} \times -\frac{\sqrt{3}}{2} + \frac{1}{2}$

$= -\frac{1}{2} + \frac{1}{2} = 0$

۲ ۲)

$\cot x = 4 \rightarrow \frac{\cos x}{\sin x} = 4 \rightarrow \cos x = 4\sin x$

$\frac{3\cos x - \sin x}{\cos x + \sin x} = \frac{3(4\sin x) - \sin x}{4\sin x + \sin x} = \frac{11\sin x}{5\sin x} = \boxed{\frac{11}{5}}$

۲ ۳)

$\sin\alpha = 2\cos\alpha$ , $\sin < 0$ , $\cos < 0$ $\cos\alpha = ?$

$\sin^2\alpha + \cos^2\alpha = 1 \rightarrow (2\cos\alpha)^2 + \cos^2\alpha = 1 \rightarrow 4\cos^2\alpha + \cos^2\alpha = 5\cos^2\alpha = 1$

$\rightarrow \cos^2\alpha = \frac{1}{5} \rightarrow \cos\alpha = \frac{1}{\sqrt{5}}\times\frac{\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{5}}{5} \rightarrow \boxed{\cos\alpha = -\frac{\sqrt{5}}{5}}$

۲ ۴)

الف) $\sin\alpha > 0$ , $\cos\alpha < 0$ , $\sin\alpha = \frac{\sqrt{2}}{10} \rightarrow \cos\alpha = \sqrt{1 - \frac{2}{100}} \rightarrow \cos\alpha = \sqrt{\frac{98}{100}} = -\frac{7\sqrt{2}}{10}$

$\cos\left(\frac{11\pi}{4}+\alpha\right) = \left(\cos\frac{11\pi}{4}\times\cos\alpha\right) - \left(\sin\frac{11\pi}{4}\times\sin\alpha\right) = -\frac{\sqrt{2}}{2}\cos\alpha - \frac{\sqrt{2}}{2}\sin\alpha$

$= \left(+\frac{\sqrt{2}}{2}\times+\frac{7\sqrt{2}}{10}\right) - \left(\frac{\sqrt{2}}{2}\times\frac{\sqrt{2}}{10}\right) = \frac{7}{10} - \frac{1}{10} = \boxed{\frac{6}{10}}$

ب) $\sin\alpha > 0$ , $\cos\alpha > 0$ , $\tan\alpha = \frac{1}{7} \rightarrow \frac{1}{49} + 1 = \frac{50}{49} = \frac{1}{\cos^2\alpha} \rightarrow \cos^2\alpha = \frac{49}{50} \rightarrow \cos\alpha = \frac{7}{5\sqrt{2}}$

$\rightarrow \cos^2 + \sin^2 = \frac{49}{50} + \sin^2 = 1 \rightarrow \sin^2 = \frac{1}{50} \rightarrow \sin = \frac{1}{5\sqrt{2}}$

$\sin\left(\frac{13\pi}{4}+\alpha\right) = \left(\cos\frac{13\pi}{4}\times\sin\alpha\right) + \left(\sin\frac{13\pi}{4}\times\cos\alpha\right) = \left(-\frac{\sqrt{2}}{2}\times\frac{1}{5\sqrt{2}}\right) + \left(-\frac{\sqrt{2}}{2}\times\frac{7}{5\sqrt{2}}\right)$

$= -\frac{1}{10} - \frac{7}{10} = -\frac{8}{10} = -\frac{4}{5}$

۵)

$2\sin^2 x+\cos^2 x=\frac{4}{3} \to \sin^2 x+\sin^2 x+\cos^2 x=\frac{1}{3}+1 \to \sin^2 x=\frac{1}{3}$

$\to \frac{1}{3}+\cos^2 x=1 \to \cos^2 x=\frac{2}{3} \to \tan^2 x=\frac{\sin^2 x}{\cos^2 x}=\left(\frac{\frac{1}{3}}{\frac{2}{3}}\right)=\boxed{\frac{1}{2}}$

۳

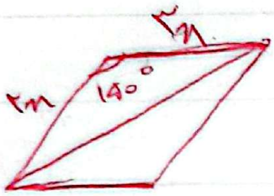
۶)

$S_{ABC}=\frac{1}{2}\times\sqrt{2}\times 6\times\sin\alpha=9\times\frac{1}{3} \to 3\sqrt{2}\sin x=3 \to \sin x=\frac{3}{3\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}}$

$=\frac{\sqrt{2}}{2} \to \alpha_{max}=120^\circ$ و $\alpha_{min}=60^\circ \to \frac{120}{60}=\boxed{2}$

۳

۷)




$S=54=2\times\frac{1}{2}\times 3x\times 2x\times\frac{1}{2} \to 54=3x^2$

$\to 18=x^2 \to x=\sqrt{18}=3\sqrt{2}$

$\to$ محیط $=2(9\sqrt{2}+6\sqrt{2})=\boxed{30\sqrt{2}}$

۴

۸)

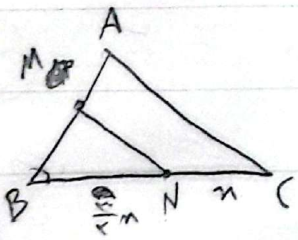
$S_{ABC}-S_{ADE}=1.75 \to 17.5\sin A-14\sin A=3.5\sin A=1.75 \to \sin A=\frac{1}{2}$

$S_{ABC}=\frac{1}{2}\times 5\times 7\times\sin A=\frac{35}{2}\sin A=17.5\sin A$ و $S_{ADE}=\frac{1}{2}\times 4\times 7\times\sin A=14\sin A$

$\to \frac{1}{4}+\cos^2 A=1 \to \cos^2 A=\frac{3}{4} \to \cos A=\frac{\sqrt{3}}{2} \to \tan A=\left(\frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}}\right)=\frac{1}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}=\boxed{\frac{\sqrt{3}}{3}}$

۳

۹)




$2BN=3NC \to BN=\frac{3}{2}NC$

$\frac{1}{2}\times AB\times BC\times\sin B=\frac{1}{2}\times BM\times BN\times\sin B\times 3$

$AB\times BC=3\times BM\times BN \to (BM+AM)\times(\overbrace{BN+NC}^{\frac{5}{2}x})=3\times BM\times BN$

$\to \frac{5}{2}x(BM+AM)=3\times BM\times\frac{3}{2}x \to 5BM+5AM=9BM \to 5AM=4BM$

$\to AM=\frac{4}{5}BM \to \frac{BM}{AM}=\frac{BM}{\frac{4}{5}BM}=\frac{5}{4}BM$

۳

۱۰)

$\frac{1}{\sqrt{\cos^2\alpha}}-\tan\alpha=\frac{1+\sin\alpha}{|\cos\alpha|}$ , $\frac{|\sin\alpha|}{\cos\alpha}=-\frac{1}{\cot\alpha}=-\frac{\sin\alpha}{\cos\alpha} \to \sin\alpha<0$

$\to \frac{1}{|\cos\alpha|}-\frac{\sin\alpha}{\cos\alpha}=\frac{1-\sin\alpha}{|\cos\alpha|} \to \frac{\sin\alpha}{\cos\alpha}>0 \to \cos\alpha<0 \to$ $\boxed{\text{ناحیه سوم}}$

۴